PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

# The Bakerian Lecture, 1974: A View of Earth and Air

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# THE BAKERIAN LECTURE, 1974

# A VIEW OF EARTH AND AIR

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[Frontispiece]

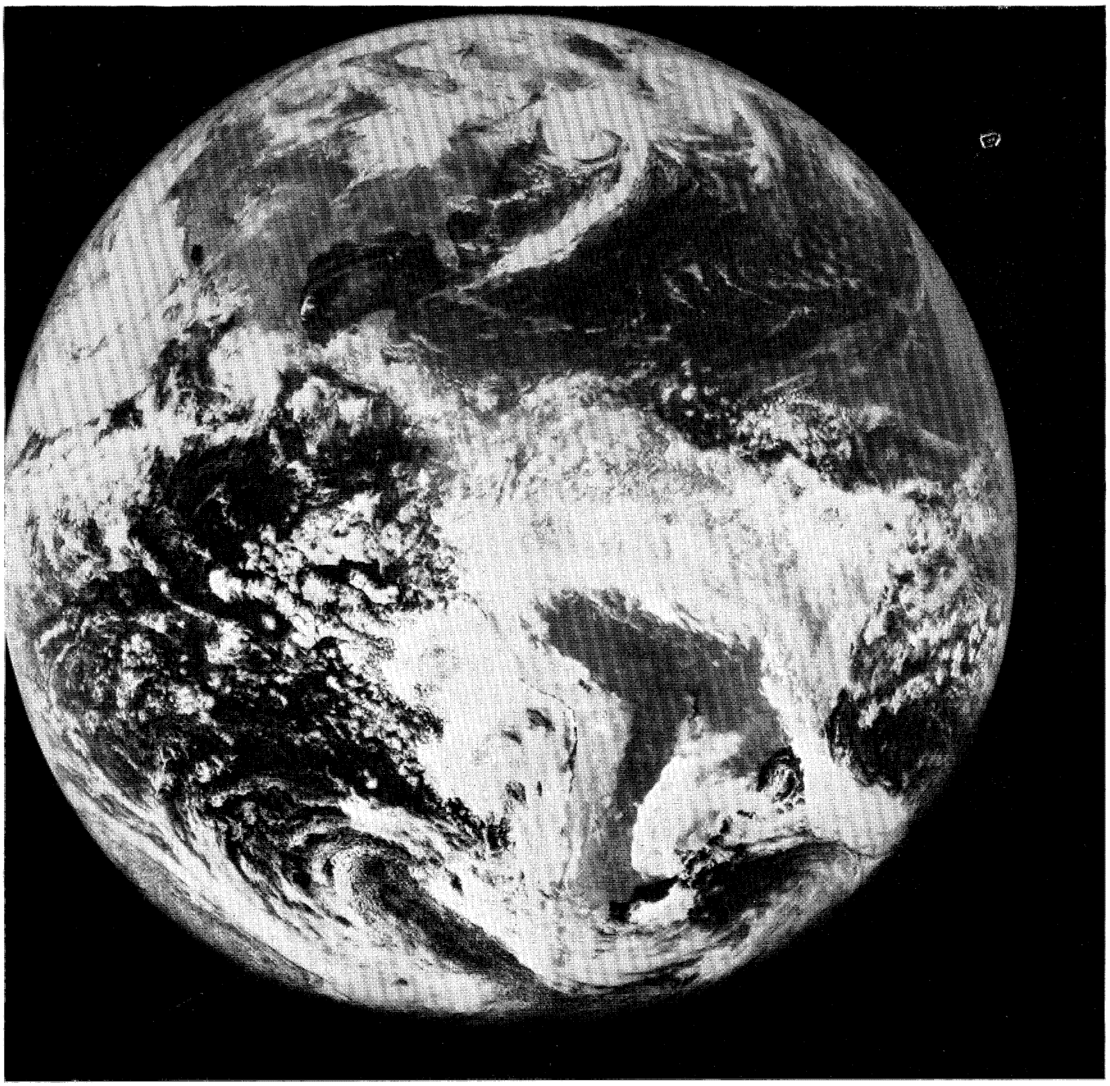
CONTENTS



My theme is the Earth and its atmosphere as viewed by an interstellar sightseer who arrives in the solar system, able to see to the limits of present human knowledge but no further, and able to turn the clock back but not forward.

This celestial snooper sees a bleeding Sun pouring out its substance as a supersonic 'solar wind', within which the magnetic Earth carves out a tadpole-like cavity, the magnetosphere, with a tail stretching millions of kilometres downwind. In the magnetosphere ionized atoms predominate from a height of 60000 km down to 2000 km; below that are layers where neutral hydrogen, helium, atomic oxygen and nitrogen are dominant in turn. Here, particularly at heights between 100 and 1000 km, analysis of the orbits of artificial satellites has revealed strong winds and huge variations in density (by a factor of up to 100) and temperature (by up to 600 K), largely controlled by the Sun. Looking back on past ideas of the atmosphere, the visitor from space notices how often scientists have forsaken correct ideas in favour of novelty.

The shape of the Earth itself has excited curiosity since the stone age, and the saga of its discovery is a microcosm of human endeavour: first seen as a sphere, measured by Eratosthenes; then fashioned by the fertile mind of Newton into a flattened sphere; and now revealed as a slightly pear-shaped spheroid with a potato-shaped equatorial section, a surface on which continents may move like rigid plates over a lake of treacle. The celestial visitor sees scientists measuring the Earth's figure correct to one metre, and soon, possibly, correct to a few centimetres by means of laser ranging to artificial satellites.



A view of Earth and Air on 28 May 1974, from the NASA Synchronous Meteorological Satellite 1 at a height of 36000 km above the equator at longitude 75° W. The southern half of the U.S.A. is free of cloud, but South America is partly hidden by clouds. The west coast of Africa can be seen on the right and storms in the Atlantic at the top right.

(*Facing p.* 67)



PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

## 1. In from afar

May I ask you to imagine that you are a being from a higher sphere, moving benevolently through interstellar space on a journey of planetary inspection? You see the Sun and its attendant planets, and you streak into the solar system to take a look at one planet in particular, the Earth and its envelope of air. I assume that the interpretation of your view of the Earth and Air can go to the limits of current human knowledge, but no further; and that you can and will turn back the clock, to see what human beings in the past thought about their planet. Reflecting on past errors is a useful reminder of the frailty of present concepts; and of the frailty of past scientists, who often abandoned correct ideas in favour of novelty. Still, this was better than slavishly accepting the word of some ancient sage, and escaping from such slavery was the aim of the founders of the Royal Society. Their motto, *Nullius in verba*, is today sometimes read cynically as 'believe in the word of no-one'. But this is not what was meant: the motto comes from two lines of Horace,

Ac ne forte roges, quo me duce, quo lare tuter:
Nullius addictus iurare in verba magistri.

(Fairclough 1961). The second line may be translated as 'In the word of no master am I bound to believe', and to those not reared on Horace the motto would have been clearer as *Nullius . . . in verba magistri*: don't slavishly believe in the word of any *master*. As I read it, the motto tells scientists to be sceptical of pompous authority, to rely on measurement rather than hearsay; but not always to reject folklore, which may sometimes be wisdom ten times distilled rather than old wives' tales. I believe that poets often express this distilled wisdom, and so penetrate the facade of appearance to see scientific truths far ahead of their time (King-Hele 1962*b*). I therefore make no apology for peppering my text with quotations from the poets.

## 2. The bleeding Sun

As you swing through interplanetary space in your role as inspector of the solar system, you will notice that the Sun is bleeding, with its substance pouring out into the vacuum around. Streams of charged particles, mostly protons and electrons, but also a few nuclei of helium and heavier atoms, are rushing past you at speeds averaging about 400 km/s (Pintér 1974); and sometimes, when the Sun unleashes one of its convulsive eruptions, the stream accelerates to 1000 km/s behind the advancing shock wave. The Sun rotates once every 27 days, so the outflowing stream revolves rather like the spray from a rotating water-sprinkler, and often like a four-jet sprinkler, because the outflow has the habit of dividing into four sectors, as shown in figure 1, with magnetic fields alternating inwards or outwards (Wilcox 1968). The sector structure may influence weather on Earth (Wilcox *et al.* 1974).

This rotating outflow of solar protons has come to be called the 'solar wind' (Parker 1958): it is a good name, and the pity is that more terms in science are not familiar four-letter words instead of pompous polysyllables. 'Wind' gives the right idea because the solar wind is full of 'keen fitful gusts . . . whispering here and there' (Keats 1817), and figure 1 is an idealized model, which needs to be jogged around to become realistic. The solar wind has rather a high speed by terrestrial standards: 400 km/s is 35 million kilometres per day, and the flow reaches the Earth in 4 or 5 days. Although fast, the flow cannot be called a 'strong' wind, because at the Earth's distance it has only 5 to 10 atoms/cm$^3$ (Burlaga 1971). Still, even at this low density, the Sun pours out nearly $10^{36}$ atoms, or 2 million tonnes, each day. This may seem a serious loss, but the total mass of the Sun is $2 \times 10^{27}$ tonnes; so at the present rate of bleeding it would last for more than $10^{18}$ years.

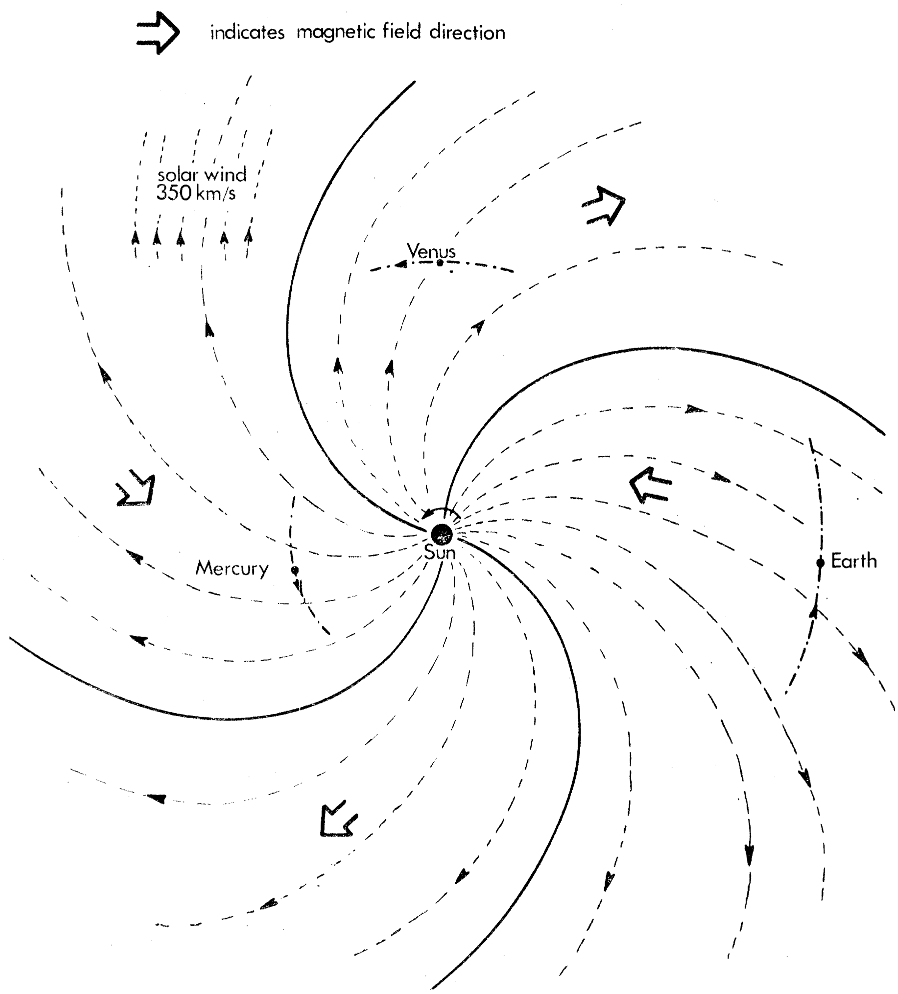



Figure 1. Sketch of trajectories of quiet solar wind particles in the ecliptic plane, with four-sector structure. (This is an idealized picture: the actual flow is irregular.)

This is the environment in which the Earth and its enveloping atmosphere have to live: a basically steady but continually twitching outflow, disturbed quite often by faster plumes of material sweeping shock waves through the steady flow, figure 2. At times near sunspot maximum, when the Sun may spit out these plumes of particles twice a week, the solar wind becomes quite chaotic. Occasionally, when fierce solar flares erupt, as on 4 August 1972, a stronger and wider plume will shoot out, with much of its energy (perhaps about $10^{24}$ J) going into a blast wave which sweeps all before it, though its impetus gradually declines as it expands (Dryer 1974). Within a day or two the declining blast wave envelops the Earth and plays havoc with the outer reaches of the atmosphere. Could the solar wind in this violent mood sweep away the atmosphere altogether, leaving the Earth to suffer the same fate as the Moon – being left naked to the bombardment of solar particles? (Catto 1974). The answer is probably 'no', or,





PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

more cautiously, 'not yet' or 'not in the last 2000 million years': the Earth's magnetic field and atmosphere have been able to carve out a tadpole-like cavity in the solar wind, as shown in figure 2, allowing parasitic life on Earth to evolve. As Erasmus Darwin (1803) expressed it,

Organic Life beneath the shoreless waves
Was born and nurs'd in Ocean's pearly caves;
First forms minute, unseen by spheric glass,
Move on the mud, or pierce the watery mass;
These, as successive generations bloom,
New powers acquire, and larger limbs assume;
Whence countless groups of vegetation spring,
And breathing realms of fin, and feet, and wing.

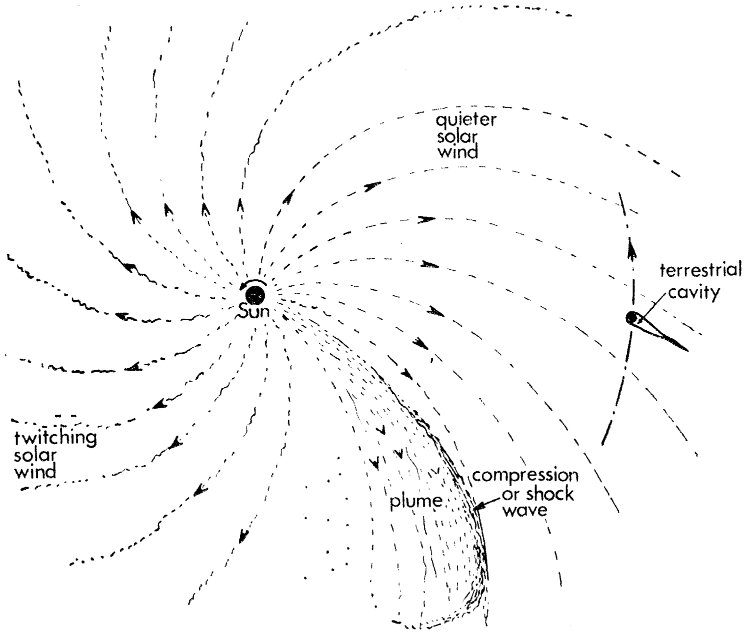



FIGURE 2. A plume of faster particles from a solar outburst disrupting the quiet (but twitching) solar wind.

Some of these parasites deem themselves very advanced, but Darwin warns them to remember that

Imperious man, who rules the bestial crowd,
Of language, reason, and reflection proud. . . .
Arose from rudiments of form and sense,
An embryon point, or microscopic ens!

But before escaping from the solar wind to enter the terrestrial cavity and inspect more closely that Earth burgeoning with life, you may like to operate your time machine, to look at some earlier ideas of the realm now called the solar wind. For many centuries Aristotle's ideas prevailed in Europe, and you are quite likely to have turned back to that era. Aristotle thought the Earth's atmosphere, or sphere of Air, had a definite upper boundary, beyond which was the sphere of Fire (Guthrie 1960). This is not a bad description of the solar wind, with its (proton) temperature of about 50000 K (Akasofu & Chapman 1972). Figure 3 shows a medieval version of Aristotle's system (Reisch 1515) from Heninger (1960). But Aristotle was wrong in making his sphere of Fire Earth-centred rather than Sun-centred, and also because he did not imagine the Sun pouring out its substance. The idea of the Sun bleeding into space is quite natural: children often draw the Sun as a bright object with rays streaming out, and the outflow is easily imagined to be more than merely light. Sun-worshippers with no written records, perhaps the Inca (Osborne 1968), may have had the idea: it is easy to imagine a solar atmosphere when the 'bright-haired Sun' hides behind thin clouds and sports a glowing aura.


Figure 3. Aristotle's view of Earth and Air, as elaborated in the sixteenth century (Reisch 1515).

The idea of a gaseous envelope extending out from the Sun certainly found favour in the eighteenth century. De Mairan (1754), in his famous book about the aurora, regarded the zodiacal light, the solar corona and auroral displays as evidence of an 'atmosphère solaire' extending to the Earth and beyond. Horsley (1767), using a fallacious method, concluded that the 'solar atmosphere', as he defined it, reached out to more than 500000 miles. Sir Richard Phillips (1818) speculated on 'whether the space extending from the Sun to the outermost planet of the Solar System be filled with a revolving solar atmosphere'. Phillips was wrong with most of his ideas, but right here.

In the late nineteenth century interplanetary space came to be looked on as a vacuum, a blank unchartable area filled only with the phantom flow of the aether. Why should the Sun bleed? Its strong gravitational pull would restrain any atoms trying to escape, and they would not succeed unless they had a speed greater than 400 km/s, which corresponds to a temperature of 600000 K, whereas the Sun's surface has a temperature of only 6000 K. However, in the late 1930s, evidence began to accumulate that parts of the corona had a temperature of $10^6$ K (Ellison 1956). This did not prove that the flow was outward: it might be that *inflowing* material colliding with the chromosphere created the high temperature. This was a popular (and not unreasonable) view in the 1950s (Hoyle 1955; Lyttleton 1956).

But before that there had been many pointers to the idea of a solar wind: the work of Störmer on the trajectories of auroral particles (see, for example, Störmer 1955); the study by Chapman & Ferraro (1931) of the impact of solar plasma on the Earth at the time of magnetic storms; and the interpretation of comets' tails by Biermann (1951). The idea of an extensive solar atmosphere was revived by Chapman (1957), and soon afterwards Parker (1958) produced a dynamic model and showed that there were several possible regimes. The material would just fall back if its energy were too low; at higher energies there could be a subsonic outflow; and it was also possible that a region near the Sun could behave like the throat of a rocket motor with the flow becoming supersonic beyond the throat and remaining at a fairly constant speed, near 400 km/s, out to beyond the Earth's orbit. Measurements from space probes have confirmed this supersonic rocket motor model (see, for example, Axford 1968; Hundhausen 1968). But, just as a rocket motor needs a fierce and rapid combustion to produce a supersonic jet, so the stoking mechanism in the corona must be right: the temperature must fall off less slowly than the inverse of the radial distance. The solar wind was therefore not obvious, and had to wait until about 1960 for official recognition. Since then the theoretical models have been improved, and the solar wind has been extensively investigated *in situ* by space vehicles (see, for example, Parker 1969; Scarf 1970). Yet it is still largely uncharted because it is so vast in scale, probably extending to beyond Pluto (Axford 1973), and also being three dimensional, not confined (like the measurements so far) to the plane of the planets. The solar wind also tends to defy systematic measurement by being so variable in its behaviour, depending on the phase of the 27 day solar rotation and the phase of the 11 year sunspot cycle, as well as being punctuated by outbursts from solar flares, and pervaded by plumes from lesser solar disturbances.

## 3. A shelter from the solar wind

The Earth's strong magnetic field, believed to be generated by dynamo action in the core, provides the creatures of Earth with shelter from the stormy blast of the solar wind, and controls the shape and size of the terrestrial cavity in the solar wind. Your all-seeing eye will register this as you come in, and by turning back the clock you will also see how strongly the magnetic field has influenced life on Earth in the past. By deflecting most of the cosmic rays the magnetic field has reduced the high-energy radiation impinging on Earthly creatures, and consequently their mutation rates. So biological evolution might have taken a different course if the magnetic field had taken different values, and the Earth might never have had to endure the ravages caused by having humans as its 'lords and masters'. The magnetic field may also influence long-term climatic variation (King 1974). If so, the magnetic field has down the centuries not only determined the size and shape of the air space essential to life on Earth, but also affected the style of human living by deciding whether peasants should grow vines rather than potatoes, for example.

The impact of solar plasma on the magnetized Earth was first studied by Chapman & Ferraro (1931), with their eyes on occasional magnetic storms rather than a continuing solar wind. But their ideas were valid for a continuous plasma, and the meeting of the approaching solar wind and the Earth's magnetic field can be seen (Martyn 1951) as a battle between the magnetic pressure $B^2/8\pi$ of the Earth's field $B$ and the total pressure of the solar wind (kinetic and magnetic), which is on average about $3 \times 10^{-11}$ Pa (Akasofu & Chapman 1972). These two forces balance at a geocentric distance of about 10 Earth radii (just over 60000 km) on the Earth–Sun line, so this is where the action is likely to be concentrated. The solar wind meets this obstacle in its path, and being supersonic, with Alfvén number often about 8 and the particle kinetic energy greater than the magnetic energy, it behaves rather like the airflow in a supersonic wind tunnel when it meets a spherical obstacle. A bow shock wave develops in front of the obstacle, at about 15 Earth radii on the sunward side. This is the fragile interface – wafer-thin by celestial standards – where the forces of heaven meet the forces of Earth:

now in little space
The confines met of empyrean Heaven
And of this World,

as Milton (1667*b*) put it in *Paradise Lost*. Behind the shock wave is a disturbed region known as the magnetosheath, and then, behind a boundary called the magnetopause, is the Earth-dominated magnetosphere (figure 4).

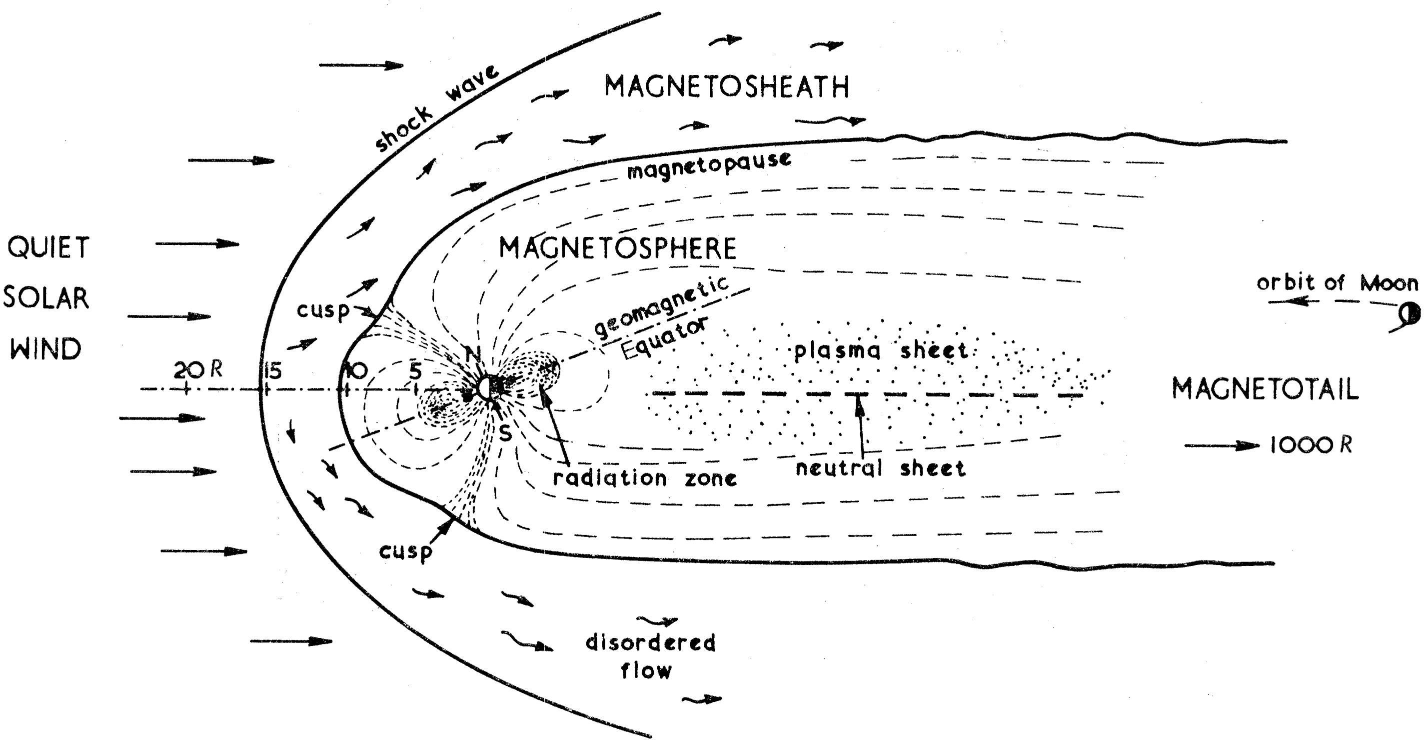



FIGURE 4. The magnetosphere and its interaction with the solar wind. – – –, geomagnetic field lines; →, particle flows; *R*, Earth's radius; ▦, regions of enhanced particle population. N and S indicate the Earth's North and South poles.

The magnetosphere is quite well defined on the sunward side up to a latitude of 60°. The Earth's magnetic field lines run from the southern hemisphere to the northern, forming closed loops up to quite high latitudes, as shown in figure 4 (cf. Choe & Beard 1974). But nearer the poles there is a narrow region known as the dayside cusp where the lines may be open and the solar-wind protons and electrons may enter (Roederer 1974).

There can be few 'spheres' less spherical than the magnetosphere, and its downstream region, the magnetotail, is drawn out to a very great distance, probably 1000 Earth radii: in shape it is not too unlike the glowing gas tail of a comet. As a structure, the tail is extremely variable and complex, and also perhaps the weakest thing in the world: with the magnetotail, science has given

to airy nothing
A local habitation and a name.

(Shakespeare 1595). The magnetotail could be blown away by a breath of fresh air, and even the variations in the tenuous solar wind (at 5 atoms per cubic centimetre) are enough to make it flap like a windsock in a terrestrial wind. Among the notable features of the magnetotail at distances beyond 10 Earth radii are the reservoir of ions known as the plasma sheet, and a neutral sheet current dividing the magnetotail into two lobes. Of the magnetic field lines extending to great distances, some may be closed and others open, connecting with the interplanetary field and letting plasma flow in (see Paulikas 1974; Wolfe & Intriligator 1970).

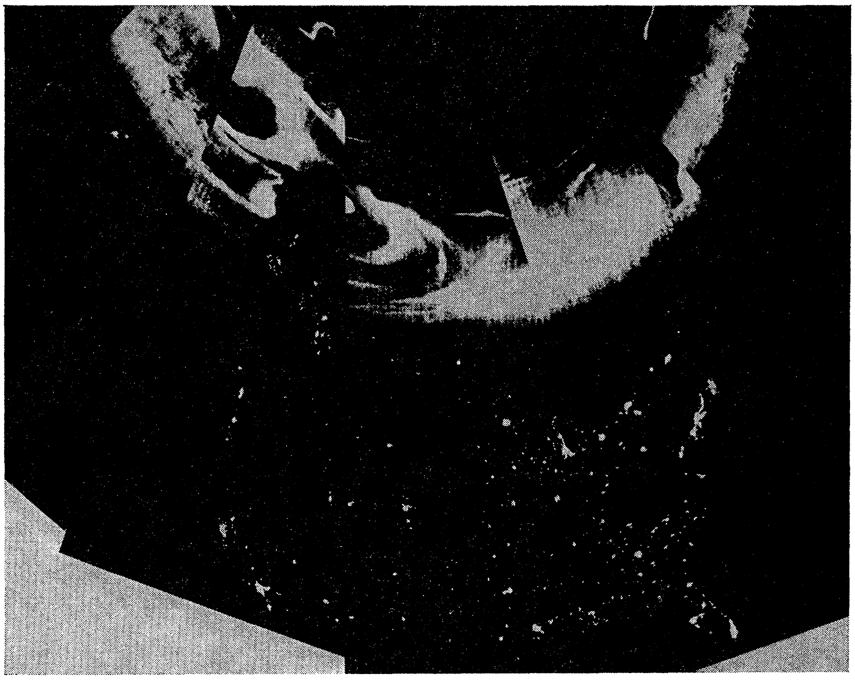


FIGURE 5. Aurora and city lights, U.S.A. and Canada, near midnight 14 February 1972. Mosaic photograph from five satellite passes: the straight lines mark the dividing lines between passes. Reproduced by permission of Dr E. H. Rogers.

This is an absurdly oversimplified picture of the quiet magnetosphere. In reality, too, like the wriggling tadpole it resembles, the magnetosphere scarcely ever is quiet: it is forever suffering disturbances or recovering from them, and may sometimes be entirely disrupted by a strong solar outburst. Then the solar particles enter quite easily, and run down the field lines to near-polar end-points on Earth, creating the fine auroral displays that have amazed the Earth's inhabitants for millions of years. They have been right to be amazed, for the rate of energy release in a major magnetic storm may be as much as $10^7$ MW, more than the world's present man-made electricity output, as is nicely shown in figure 5, which is a satellite mosaic photograph of the United States and Canada by night. An aurora of moderate intensity over

Canada easily outshines all the neon lights. Even the quiet solar wind probably transfers about $10^6$ MW continuously to the magnetosphere (Roederer 1974), quite enough to explain the unresting activity of the magnetosphere, and of the upper reaches of the neutral atmosphere lying below.

Before you leave the magnetospheric plasma to sample lower levels, you may care to see some previous views of the magnetosphere. Aristotle did quite well with his universe of geocentric spheres – the central sphere of Earth, the thin spherical shell of oceanic water, and then the huge sphere of Air with a definite upper boundary, identifiable with hindsight as the magnetopause; the highest region was very hot (Lee 1952) from contact with the sphere of Fire outside. This picture, figure 3, remained in favour until the seventeenth century. But it was without factual foundation and was condemned by the precept *Nullius in verba* [*magistri*], especially since the *magister* was Aristotle. Instead it was assumed, until about 1950, that the atmosphere just merged imperceptibly into the blank of interplanetary space, with no recognizable transitional structure. This was admirably scientific, but unfortunately also wrong.

Scientists also need to be a little humble about the idea that the atmosphere might extend to the Moon, a highly unscientific notion popular in many cultures. For example, Seleucos the Chaldean in the second century B.C. explained the tides in the Persian Gulf by the resistance of the Moon to the rotation of the atmosphere (Needham 1954). In the second century A.D. Lucian of Samosata wrote a story about mariners who were carried to the Moon by an upblast of atmospheric air (Harmon 1961). In the seventeenth century Francis Godwin wrote about a gentleman called Gonsales who trained a flock of geese to tow him through the air: he didn't know it, but the geese hibernated on the Moon, and they took him there (Godwin 1638). More impressive, because it comes so close to modern views, is the definition of the magnetotail by Milton (1667*a*) in *Paradise Lost*. He tells us how certain foolish clerics are blown by 'a violent cross-wind' for 'ten thousand leagues [about 50000 km] into the devious Air'. Thus 'upwhirld aloft', they

Fly o'er the backside of the World farr off
Into a *Limbo* large and broad, since calld
The Paradise of Fools

– presumably so called because no one with any sense would wish to go there.

Scientists in the past hundred years have often laughed at the idea of the atmosphere extending to the Moon, but they have had to stop laughing now they know that the Moon can come inside the Earth's magnetotail, for up to 3 days each month when the geometry is right. Shelley's love-duet of the Earth and Moon in *Prometheus Unbound* has been admired for its picture of tidal rotational locking and nutational rocking (King-Hele 1971):

THE MOON: I, a most enamoured maiden
Whose weak brain is overladen
With the pleasure of her love,
Maniac-like around thee move
Gazing, an insatiate bride,
On thy form from every side. . . .

(Shelley 1820*b*). Other lines, previously thought fanciful, can be read as factual now it is known that a diaphanous flow of Earth-born atoms caresses the full Moon's face:

THE MOON: Brother, wheresoe'er thou soarest
I must hurry, whirl and follow
Through the heavens wide and hollow,
Sheltered by the warm embrace
Of thy soul from hungry space.

## 4. Down into thin air

Now, returning from time travel to space travel, you come down from the high vacuum of the magnetosphere, where magnetic fields and plasmas reign supreme, to begin breathing the thin air of the upper atmosphere, where neutral atoms are predominant. Figure 6 shows the major components at heights below 3000 km.

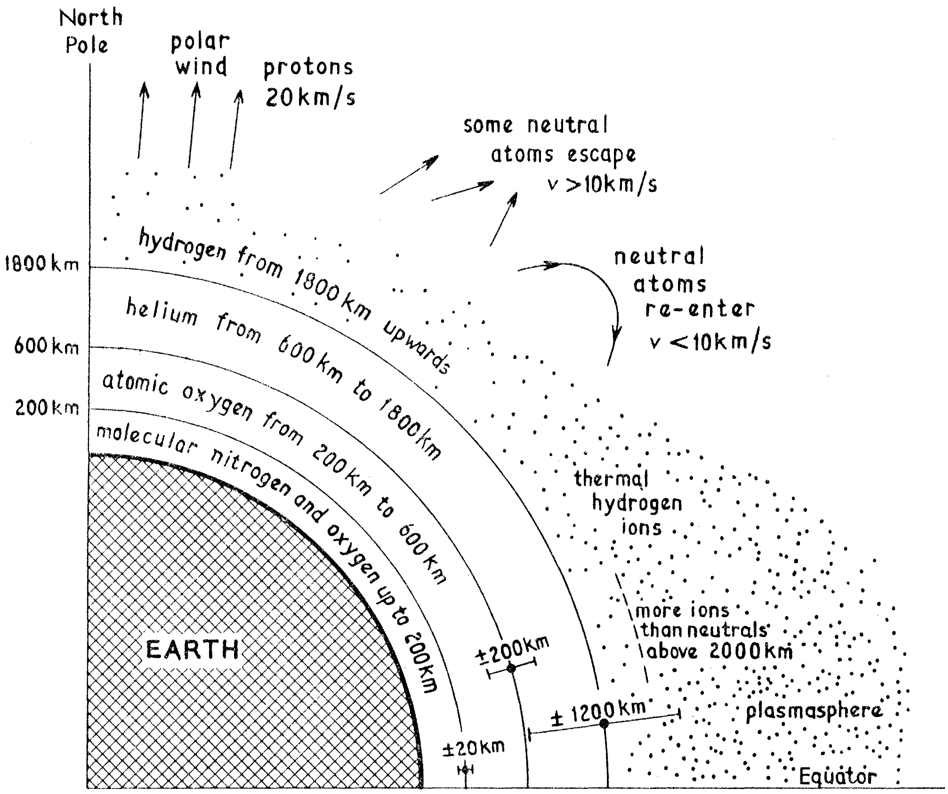



FIGURE 6. Major atmospheric components, for medium solar activity: temperature 900 K in upper thermosphere. Not to scale and much oversimplified. ⊢•⊣, Variation in boundary during a sunspot cycle (boundary is lower for low solar activity).

As you descend, you look first at heights from about 3000 km down to about 1000 km. At the top of this height-band the ions outnumber the neutral atoms, while lower down the neutral atoms predominate. The ions near the upper height limit, controlled by the magnetic field, form a plasmasphere of protons at normal temperatures ranging over low and middle latitudes, and this is also the home of the inner radiation zone of higher-energy protons, resulting from albedo neutron decay (White 1973). But near the poles, with the magnetic field lines almost radial, there is probably an outflow of protons, called the 'polar wind', which is like a feeble version of the solar wind, and has a supersonic outflow at heights above 2500 km, at a speed of about 20 km/s. The loss of hydrogen, though it gives oxygen ions dominance to a greater height, is not too serious because it probably only amounts to about 1 g $km^{-2}$ $day^{-1}$, much of which may be restored by inflow from the solar wind. It should be emphasized that, for the ions, figure 6 is a highly oversimplified sketch: the kinetics and composition of the ion exosphere are complex, with great variations dependent on latitude and temperature. See Lemaire & Scherer (1974) for a full review.

The neutral atoms in this region from 1000 to 3000 km, mainly hydrogen in the upper parts, behave quite differently from the ions. The neutral atoms are unaffected by the magnetic field and travel great distances between collisions. They often move freely in extended ballistic trajectories before returning to their homes in the region below 1000 km, there to join the disciplined chaos of kinetic collision, or be re-ejected into new trajectories, rather like celluloid balls dancing on a fountain of water. The speeds of the atoms range widely, and the high-energy tail of the distribution, with velocity greater than 10 km/s, escapes from the Earth. The proportion escaping is controlled by the temperature below 1000 km, and the equipartition of energy keeps the hydrogen atoms moving faster than the heavier helium atoms, so that hydrogen is more likely to escape. The escape of the light atoms in the high-energy tail was hinted at by Waterston (1846) and Stoney (1868). The theory of the escape was developed by Jeans (1925) and has been extended by Spitzer (1952) and Chamberlain (1963); a historical review is given by Chamberlain, and by Lemaire & Scherer (1974).

As you go down, you find that hydrogen yields its pride of place to helium at a height of about 1800 km for medium solar activity; but the transition height is critically dependent on solar activity and may vary by about ±1200 km in the course of the solar cycle, as shown in figure 6. On going down farther, helium gives way to atomic oxygen at a height of about 600 km for average solar activity, with ±200 km variation; while from heights near 200 km down to ground level, the major components are the molecular nitrogen and oxygen familiar to the inhabitants of Earth.

So the intuitive idea of the lighter gases becoming dominant at greater heights proves to be correct. There are at great heights layers of helium and then hydrogen:

> Where lighter gases, circumfused on high,
> Form the vast concave of exterior sky,

as Erasmus Darwin (1791) neatly expressed it.

## 5. Down into denser air: below 1000 km

The lowest layer of the Earth's envelope of air, below 1000 km, is best viewed in terms of its temperature, as in figure 7. Starting from the ground and going up into the troposphere, you find the average temperature falling fairly steadily from 290 K at sea level to about 220 K at a height near 10 km. The temperature stays fairly constant in the stratosphere up to about 25 km, and then rises to a maximum of 270 K at a height of 50 km, where ozone absorbs solar ultraviolet radiation and shields the creatures of Earth from its shrivelling effects. Above 50 km, the temperature decreases again in the upper mesosphere to a minimum of 180 K at a height of 85 km. Above 90 km, the temperature starts to increase sharply as solar extreme ultraviolet radiation is absorbed, and this region is aptly called the thermosphere. The temperature increases to a steady value, which may be anywhere between 600 and 1200 K, at a height near 300 km, and continues constant up to the height where temperature ceases to be a meaningful term – somewhere between 500 and 1000 km, where the neutral molecules rarely collide and their random motion is gradually transformed into an ill-directed fusillade of miniature ballistic missiles. The variations in temperature are given in more detail in the Cospar International Reference Atmosphere 1972 (*CIRA* 1972).

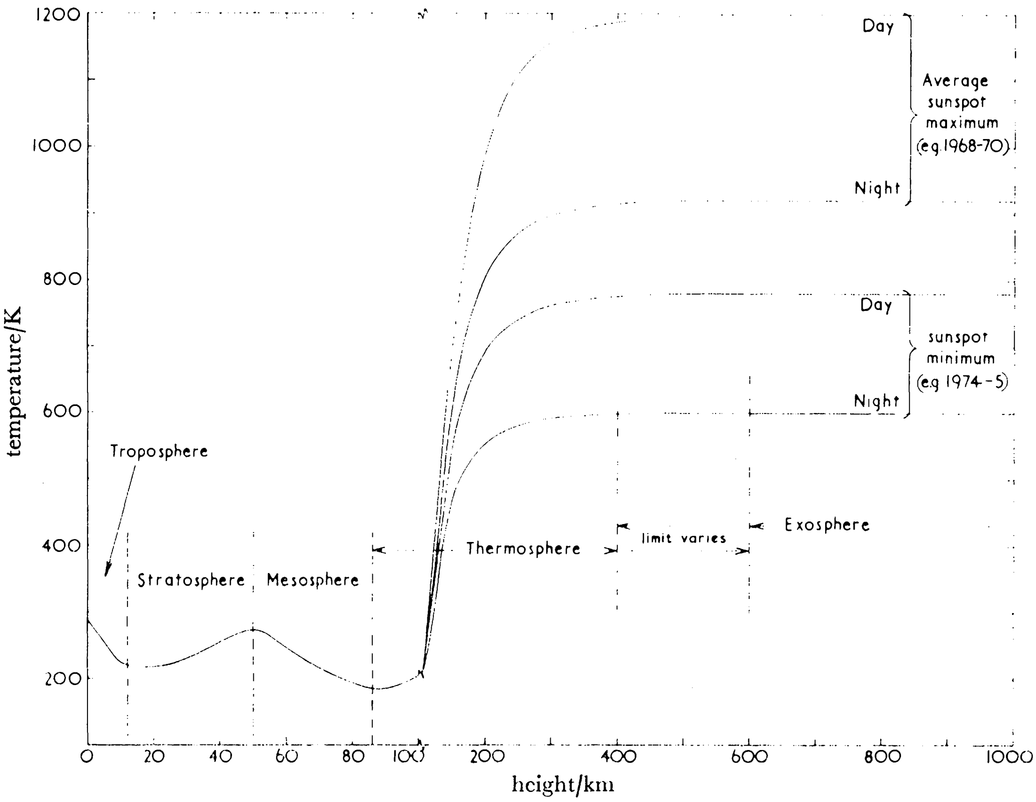



FIGURE 7. Average air temperature from sea level to 1000 km, from *CIRA* 1972.

Since the temperature of the thermosphere is nearly independent of height above 300 km, the idea of the lighter gases rising to greater heights can be expressed not only as an intuition but also, to a first approximation, in simple mathematical form. If hydrostatic equilibrium prevails, the rate of decrease of pressure $p$ per unit of height $y$ is equal to the weight of air (density $\rho$) in a unit cube, so that

$$dp/dy = -\rho g, \tag{1}$$

where $g$ is the acceleration due to gravity. Equation (1) may be rearranged as

$$dp/p = -(\rho g/p)\,dy. \tag{2}$$

But for a gas of molecular mass $M$ and temperature $T$, the gas law gives

$$p/\rho = RT/M, \tag{3}$$

where $R$ is the gas constant (8.31 J $K^{-1}$ $mol^{-1}$). Thus equation (2) becomes

$$\frac{dp}{p} = -\frac{Mg}{RT}dy = -\frac{1}{H}dy \quad \text{say,} \tag{4}$$

where

$$H = RT/Mg \tag{5}$$

is known as the 'scale height'. If $T$ and $M$ are constant and the gradual variation of $g$ with height is ignored, $H$ is constant and equation (4) can be integrated to give

$$p = p_0 \exp(-y/H), \tag{6}$$

where suffix 0 denotes values at $y = 0$. Also, since $p/\rho$ is constant,

$$\rho = \rho_0 \exp(-y/H). \tag{7}$$

So, if the temperature and molecular mass are nearly constant, as in the stratosphere for example, the density decreases exponentially with height, the density scale height $H$ – the height in which the density falls off by a factor of 2.72 – being about 7 km in the stratosphere.

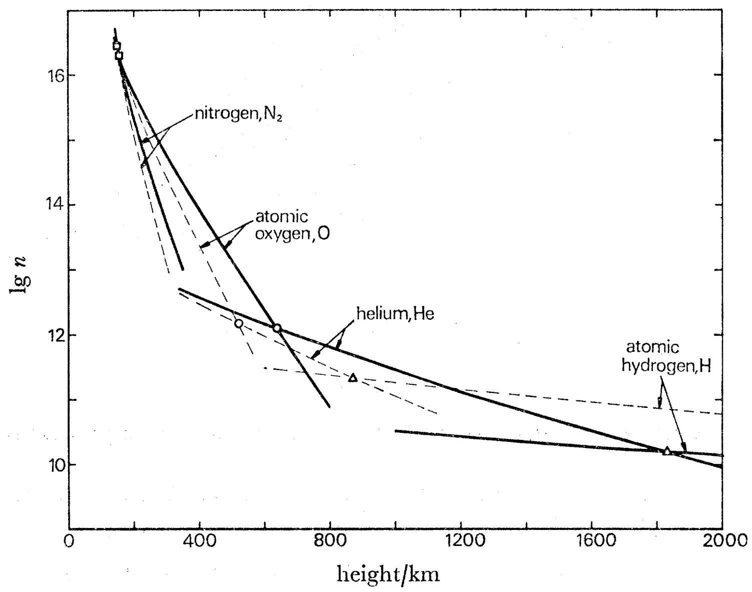



FIGURE 8. Fall-off in number density of $N_2$, O, He and H as height increases, for temperatures of 900 K and 700 K, from *CIRA* 1972. $n$ is number of atoms/$m^3$. ——, 900 K; – –, 700 K; □, $n(N_2) = n(O)$; ○, $n(O) = n(He)$; △, $n(He) = n(H)$.

In the thermosphere above 300 km, where temperature varies little with height, each individual component of the air enjoys its own scale height, $H_i$ say, inversely proportional to its molecular mass $M_i$ as indicated by equation (5), ignoring winds and other departures from equilibrium. So, to a first approximation, the logarithm of the number density $n$ for each constituent decreases linearly as height increases, with a slope proportional to $M_i$. Figure 8, based on the Cospar International Reference Atmosphere 1972, shows how the lighter components decrease more slowly as height increases, and so outnumber the heavier components at the greater heights. (The assumptions underlying equation (7) are not strictly valid in the exosphere, above 600 km, but the same trends continue there.)

The solid lines in figure 8 apply for a thermospheric temperature of 900 K, which is the average over the solar cycle. But the slope of lg *n* is inversely proportional to the temperature, from equation (5), and the broken lines in figure 8 give the values of *n* for a lower thermospheric temperature, namely 700 K, which is the daily average at sunspot minimum. Figure 8 clearly shows the great variations in the height at which a particular species begins to dominate, as figure 6 has already indicated. Figure 8 also shows how higher temperatures lead to much higher densities, particularly at heights between 500 and 1000 km.

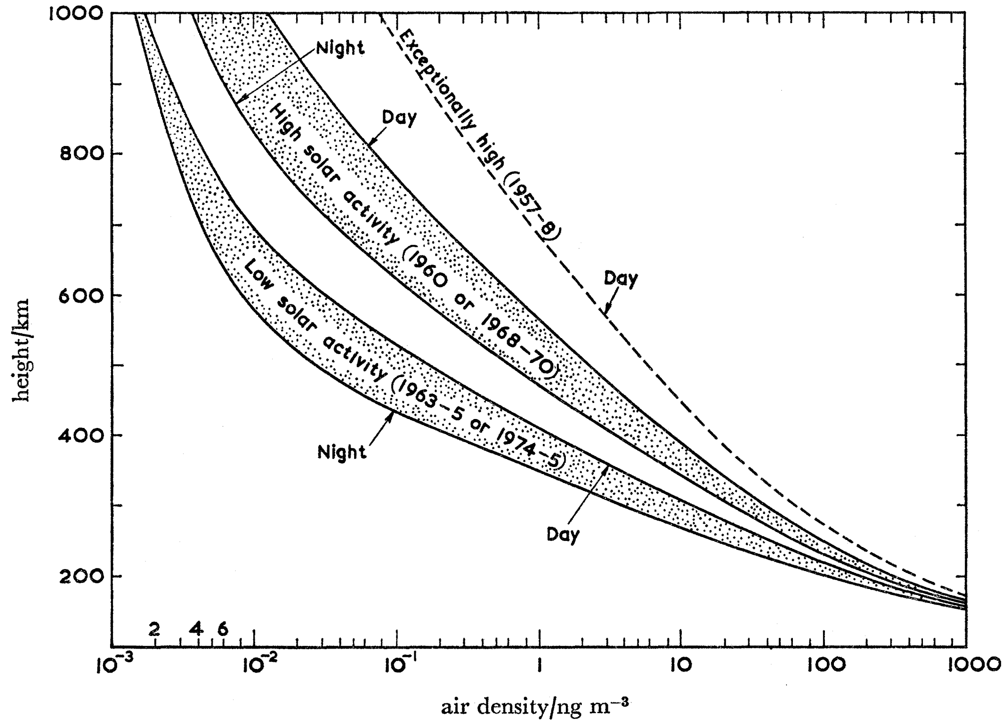



FIGURE 9. Density against height from 150 to 1000 km for low and high solar activity.

Figure 9 shows the variation of density with height, as revealed by analysis of the decay of satellite orbits between 1960 and 1970 (see, for example, King-Hele & Walker 1961, 1971; Jacchia & Slowey 1967; Priester, Roemer & Volland 1967; Marov & Alpherov 1972). As you go down from 1000 km to 150 km, the density increases by a factor of up to $10^6$: but throughout this huge range of values of density the sure guideline is the temperature – or at least the 'temperature', in quotes because its usual definition no longer applies above 600 km, and even at lower heights is modified by dynamic effects. Throughout the height range this thermospheric temperature is controlled by the Sun, and particularly solar ultraviolet radiation, which is absorbed in the lower thermosphere at heights between 100 and 200 km. The variations in this radiation during the sunspot cycle can alter the average daily temperature by 400 K, as figure 7 shows, corresponding to variations in density by a factor which increases from about 2 at 250 km to a maximum of 20 at 600 km, decreasing slowly at greater heights. The density, like the temperature, is greatest when the Sun is most active.† The presence or absence of solar

† In figure 9 the 'high solar activity' corresponds to temperatures about 100 K higher than for 'average sunspot maximum' in figure 7. The 'low solar activity' of figure 9 is the same as the 'sunspot minimum' of figure 7.

radiation has a smaller but still important effect: the maximum daytime temperature in the thermosphere exceeds the minimum by about 30% (Jacchia 1971). The maximum daytime density, attained at about 14h local time, exceeds the minimum night-time density, attained at about 04h local time, by a factor which (for average solar activity) increases from 1.5 at 200 km to a maximum of about 6 at 600 km and then decreases. At sunspot minimum the greatest day-to-night variation shifts to a lower height, and at sunspot maximum to a greater height. Every day, as regularly as a clock, the thermospheric air suffers this drastic variation. Figure 9 shows that, when the sunspot-cycle and day-to-night effects are combined, the density at sunspot maximum by day may exceed that at sunspot minimum at night by a factor of up to 100 at heights near 600 km.

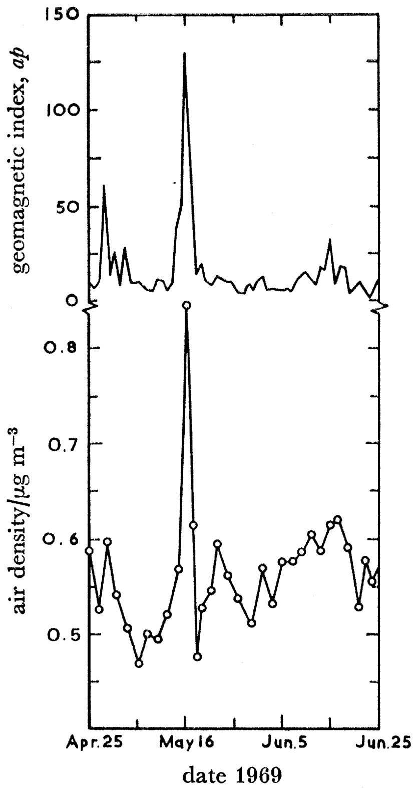



FIGURE 10. Increase in density at 180 km height at the time of a magnetic storm, from King-Hele & Walker (1971). The index *ap* measures the degree of disturbance of the geomagnetic field.

The Sun also influences the thermospheric temperature through the incoming particles of the solar wind, most obviously when the Sun erupts with flares and ejects plumes of high-energy ions which create geomagnetic storms and auroral displays on Earth. The Sun rotates once every 27.3 days, and consequently all the solar-particle effects have a 27 day recurrence tendency. When the high-energy particles from the Sun impinge on the Earth, the upper atmosphere is heated in convulsive ripples (Volland & Mayr 1971), and the temperature sometimes rises by as much as 500 K for a few hours (Jacchia 1971) with a corresponding increase in density by a factor of up to 6 at heights near 600 km. Even at a height of 180 km, where the atmosphere is relatively insensitive to solar activity, the density can increase by a factor of nearly 2 at the time of a magnetic storm, as figure 10 shows.

Apart from these major disturbances due to the Sun, the student of aeronomy has to grapple with a host of other phenomena, which can scarcely be called minor when they may double the density. Perhaps the most important of these is the semi-annual variation, so called because the density exhibits a minimum in mid-January every year, rises to a maximum in April, decreases in May towards a deeper minimum in late July, and then rises to a maximum in late October, usually higher than that in April (Jacchia, Slowey & Campbell 1969). This effect occurs at all heights between 100 and 1000 km, and the maximum density exceeds the minimum by a factor of about 1.5 at 200 km, increasing to 2.5 at 500 km and decreasing to about 2 at 1000 km (Cook 1969). The variation is particularly clear at heights near 1000 km when solar activity is low (Cook & Scott 1966), but it is also readily apparent at lower heights when solar activity is higher: figure 11 shows variations at a height of 230 km in 1970–71 (Walker 1974). The strength of the semi-annual variation is not directly related to solar activity, but varies from year to year, perhaps quite irregularly (Voiskovskii *et al.* 1973), or perhaps exhibiting a recurrence period of about 3 years (King-Hele & Walker 1969; Cook 1972). The semi-annual variation most probably arises from a seasonal variation at heights below 100 km, increasing in amplitude as it rises into the more rarefied air (Volland 1969).

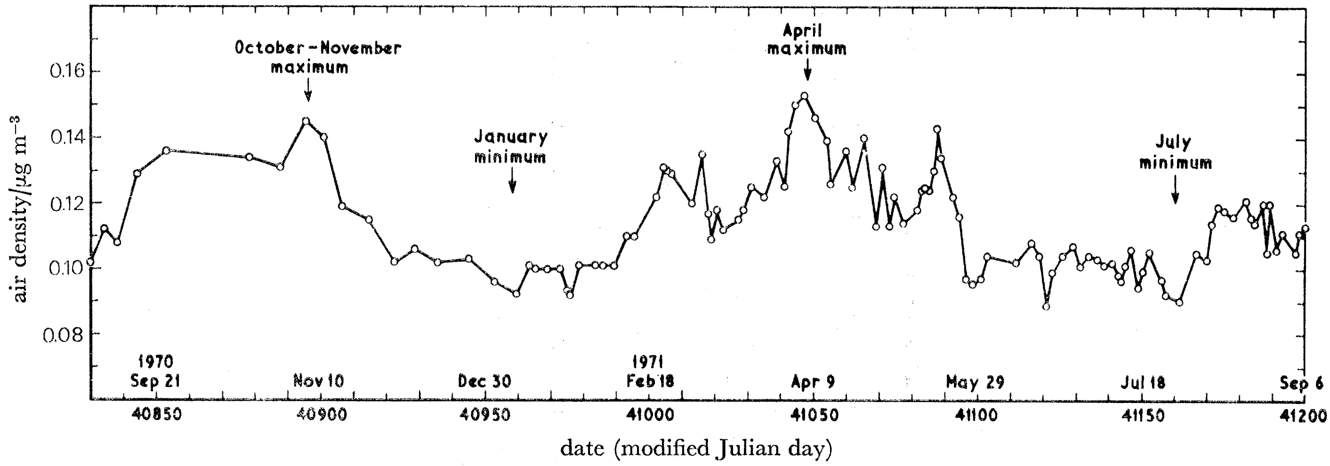



FIGURE 11. Variation of air density at 230 km height, corrected for solar-activity and day-to-night effects, showing semi-annual variation. From analysis of the orbit of 1970-65D (Walker 1974).

Many other important features of the upper atmosphere have been silently ignored in this rapid review. For example, there are the gravity waves which sweep through like an ocean swell but are necessarily transient (Hines 1972). A more important neglected feature is the region of enhanced ionization in the lower atmosphere, called the ionosphere (Rishbeth & Garriott 1969; Ratcliffe 1972): the ionosphere is even more complex than the neutral atmosphere, and has taxed the ingenuity of several generations of ionospheric physicists. But even at the maximum of the F layer, near 300 km height, less than 1 % of the molecules are ionized. Your journey through the hostile environment of interstellar space has desensitized your eyes to the presence of low-energy ions, so the ionosphere looks like a minor constituent of the atmosphere: but it is very important for the Earth dwellers because it controls the fate of radio waves. Figure 12 shows the variation in density from ground level to 1000 km for the two extremes – high solar activity by day and low solar activity at night – the ionized fraction (Johnson 1965) being indicated by the broken line up to 500 km, above which a comparison on the basis of densities is misleading since the main component is not the main ion.

Looking back on past ideas of the upper atmosphere at heights between 10 and 1000 km is not very illuminating, because so many of the ideas were wrong. But some were right, and Edmond Halley deserves an honourable mention because in 1686 he correctly calculated how air density and pressure decrease with height (Halley 1686; Armitage 1966), and deduced that the air was reduced to $\frac{1}{3000}$ of its ground-level density at a height of 41 miles (66 km): the correct height is 36 miles (58 km), so he was not far wrong. Halley's calculation was helpful in leading eighteenth-century scientists towards atmospheric models that were at least partially valid, for example Erasmus Darwin's three-layer atmosphere with a top layer of hydrogen (King-Hele 1973). Aurorae were also correctly interpreted in the eighteenth century, thanks to the idea of Halley (1716) that they were linked with the Earth's magnetic field, and the observations of glowing electrical discharges in rarefied gases (Watson 1752). Erasmus Darwin (1791) could rightly tell his many readers that auroral particles

Dart from the North on pale electric streams,
Fringing Night's sable robe with transient beams.

De Mairan (1754) was basically correct in seeing the solar atmosphere as the origin of auroral particles, and he also gave realistic values for the heights of aurorae (between 100 and 1000 km).

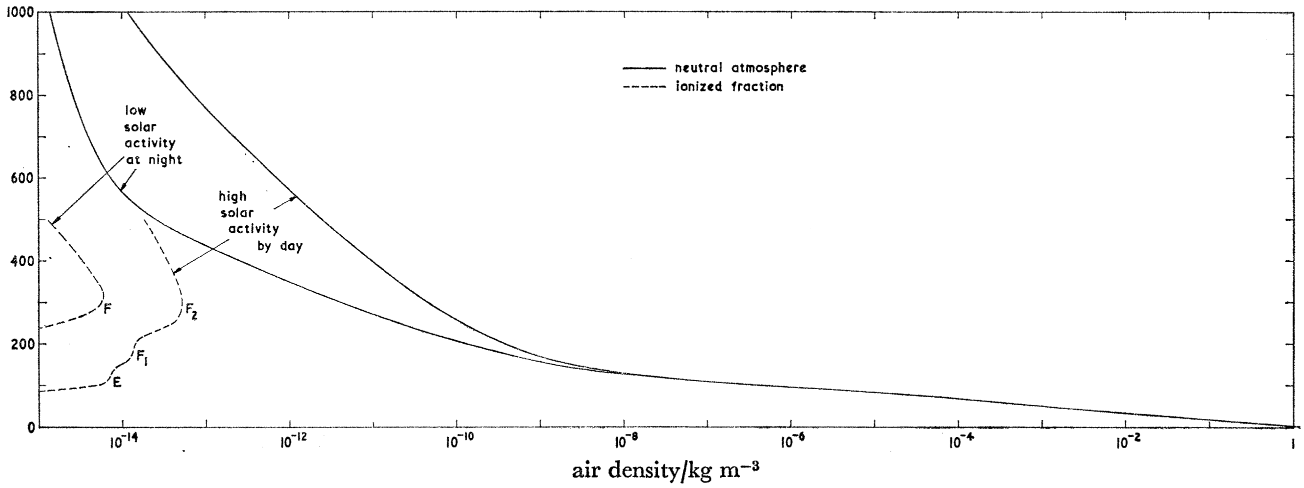



FIGURE 12. Air density from sea level to 1000 km.

The meteorologists of the early twentieth century were rather reluctant to discuss the upper atmosphere; when they had to, their assumptions were mere extrapolations. The scientists of 50 years ago assumed that the low temperatures in the stratosphere continued into interplanetary space: for example, when Jeans (1925) developed his classical theory of the uppermost atmosphere in the 1920s, he assumed, like everyone else, that the temperature remained constant at −54 °C (219 K) from the stratosphere outwards, in startling contrast to the value of 900 K now taken as the average in the thermosphere.

Even in 1957, just before the satellite launchings, the standard atmospheres in use were seriously wrong. The A.R.D.C. model (Minzner & Ripley 1956) gave a density of 26 ng/m³ at a height of 240 km, whereas the first values obtained from the observed decay rate of *Sputnik* 1 gave about 200 ng/m³ (R.A.E. 1957), a value later refined to 180 ng/m³ (King-Hele &

Walker 1960), or 7 times greater than the model. It is only fair to add, however, that solar activity late in 1957 was the highest ever recorded: with average solar activity, the density at 240 km is only about 80 ng/m$^3$, or 3 times greater than in the pre-satellite model. Still, even a factor of 3 is an error not to be lightly dismissed.

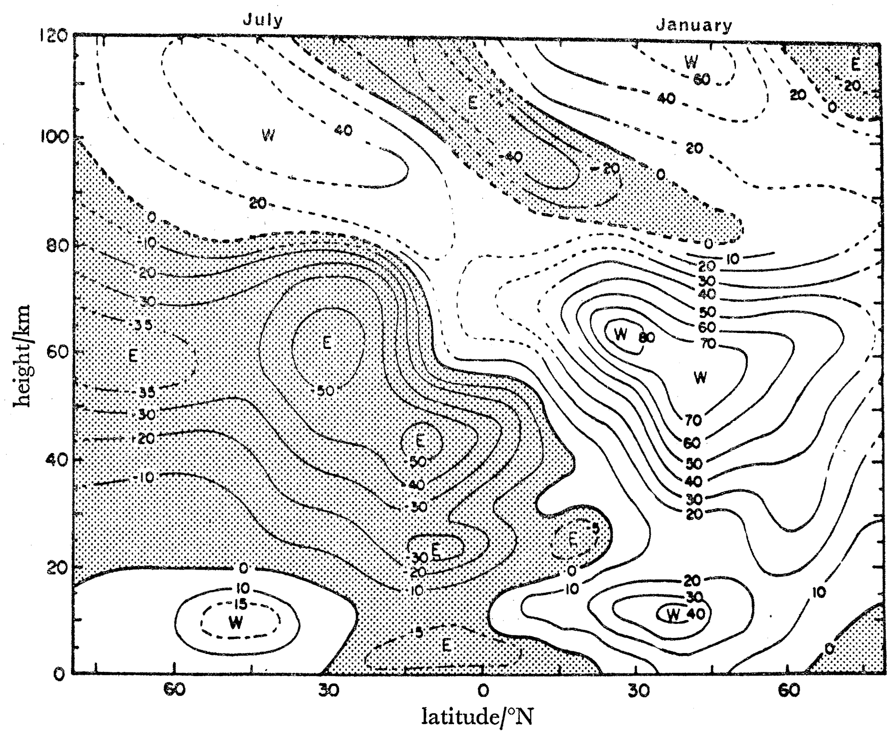



FIGURE 13. Mean January and July zonal winds up to 120 km, from *CIRA* 1972. Unshaded area indicates west-to-east, shaded area east-to-west winds. Numbers on the contours show speeds in m/s.

## 6. 'The motions of the viewless winds'

So far you have been looking chiefly at the temperature and density of the atmosphere: but the wide variations in temperature and pressure, coupled with the Earth's daily rotation, generate rapid movements of the air, which at heights above 100 km are inhibited in complex ways by the reluctance of the charged particles to cross magnetic field lines.

Up to a height of 100 km the wind patterns, although complex, are quite well charted (Groves 1969). Figure 13 shows the average west-to-east winds for January and July, from the Cospar International Reference Atmosphere 1972 (*CIRA* 1972); but this average is modulated by tidal components (12 h and 24 h) (Groves 1974), and also by the 'quasi-biennial' stratospheric winds (periods of 2–3 years).

At heights above 100 km the situation is probably more complex still, and certainly more obscure. The winds probably depend on height, latitude, season, local time, and longitude, as well as influxes of particles from the solar wind and transient disturbances in the lower atmosphere propagating upwards. Present methods of measurement are imperfect. Local measurements made by using vapour trails from rockets (see, for example, Rees, Roper, Lloyd & Low 1972) are, because of their expense, too few in number to distinguish these many effects. Analysing the changes in satellite orbits yields good average values, but significant variations are averaged out. Radar back-scatter measurements (Evans 1972) are powerful, but limited in geographical coverage and sometimes open to question over interpretation. Theoretical analyses are bedevilled by the extreme complexity of the problem and doubts over the assumptions; none of the theories produced has commanded universal assent (Rishbeth 1972). So the interpretation of atmospheric motions is still almost as mobile as the atmosphere itself, and only a few major features can be mentioned.

The first such feature is the order of magnitude of the wind speeds, and how they were found. The air drag on a satellite in orbit is directed tangential to its velocity relative to the air, and, because the atmosphere rotates, this direction differs appreciably – by up to about 5° – from that of its velocity relative to the Earth's centre. Consequently the satellite feels a sideways

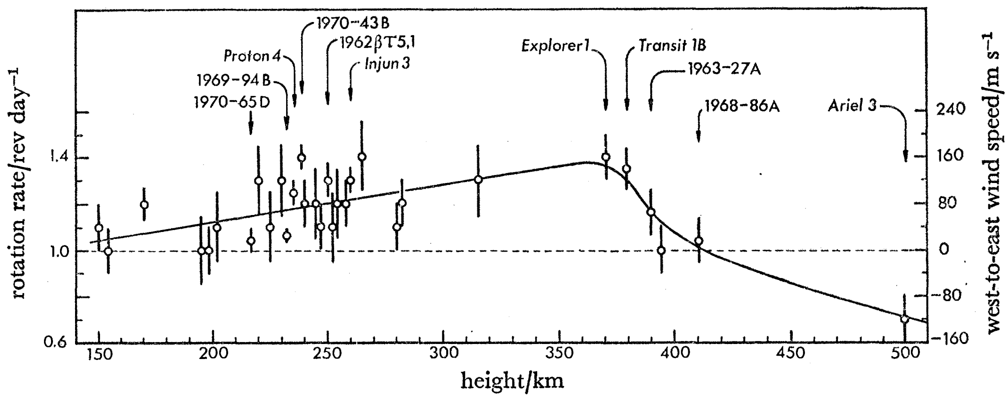



FIGURE 14. Mean upper-atmosphere rotation rate from analysis of 34 satellite orbits.

force, which has the effect of slowly reducing the inclination of the orbit to the equator. By measuring the decrease in inclination as the orbit decays, the atmospheric rotation rate near the satellite's perigee can be found. Everyone expected co-rotation, i.e. a rotation rate close to 1.0 rev/day. But the first good measurements, on *Sputnik 2* (Merson, King-Hele & Plimmer 1959), gave a value near 1.25 rev/day (King-Hele 1962*a*), thus indicating that the atmosphere at a height near 250 km was on average rotating faster than the Earth, with a west-to-east wind speed of order 100 m/s. In contrast, most theoretical studies in the 1960s suggested average zonal winds of order 10 m/s only (Kohl & King 1967; Bailey, Moffett & Rishbeth 1969). As further satellite orbits were analysed (King-Hele 1964; King-Hele, Scott & Walker 1969; Ching 1971; Hiller 1974), the results confirmed that the upper atmosphere experiences substantial 'super-rotation'† at heights of 200–350 km. Figure 14 shows results from 34 satellites, which indicate that the average rotation rate increases from 1.0 rev/day at a height of 150 km to nearly 1.4 rev/day at 350 km, though the scatter suggests considerable departures from the average. Above 400 km the rotation rate decreases, as would be expected because this is the region where molecules pursue individual trajectories: the molecules circling fastest are those most likely to escape, thus lowering the average speed. The rotation rate probably falls to about 0.8 rev/day by 500 km height (Gooding 1971; Prior 1972).

With the meridional (north-to-south) winds, some regular tidal components have been measured (Amayenc 1974): the pattern is less complex than for the zonal winds, with speeds again of order 100 m/s.

† Although semantically improper, 'super-rotation' has the great virtue of being easily understood.

The wind speed of 100 m/s that emerges as typical implies that dynamic heat transfer has an important influence on the energy balance of the upper atmosphere. Locally, however, winds may vary widely from the average, and measurements of the motions of vapour trails released from rockets at high latitudes have revealed winds of up to 500 m/s at heights of 120–150 km at times when disturbances in the solar wind have excited auroral activity (Rees 1971). Even stronger winds, up to 1100 m/s, have been recorded at heights near 150 km by accelerometers aboard a satellite that sampled the auroral region during a severe magnetic storm (Feess 1973). Recent theoretical studies also indicate the possibility of very strong winds (Izakov, Morozov & Yashchenko 1973; Chang, Wu & Smith 1974).

The idea that the upper atmosphere might go round faster than the Earth was thought rather shocking in the early 1960s: many scientists had somehow become firmly attached to the idea that the atmosphere was firmly attached to the Earth,

Rolled round in Earth's diurnal course
With rocks, and stones, and trees.

(Wordsworth 1800). To have to question a belief of such comforting stability was disturbing; but if you turn back the clock, you will find many earlier theorists not addicted to co-rotation. Aristotle, and the many generations of scholars who followed his teaching, thought the Earth was non-rotating while the upper atmosphere was dragged round by the friction of the rotating higher spheres of the fixed stars (Lee 1952), thus implying a 400 m/s east-to-west wind. Shelley, the most aerial of poets, liked to think of the Earth's rotation as being driven by the atmospheric rotation,

those streams of upper air
Which whirl the Earth in its diurnal round.

(Shelley 1824). So, in his view, the Earth is lagging behind the more rapidly rotating upper atmosphere. Curiously enough, he even chose the correct height for the phenomenon: he is describing the antics of the Witch of Atlas, and she went up to the level where super-rotation occurs after streaking around at heights between 50 and 100 km, where

She ran upon the platforms of the wind,
And laughed to hear the fireballs roar behind.

Quite apart from these chance hits from the past, there are several earthly and unearthly reasons for not being surprised at super-rotation. At the down-to-earth level the atmosphere does super-rotate slightly on the surface, with a mean west-to-east wind of 1 m/s or more; and also at stratospheric level, where the west-to-east jet streams whirl round. The best unearthly example is Venus, whose atmosphere rotates in about 4 Earth days, and therefore makes about 60 revolutions per Venusian day – hyper-rotation *par excellence*. The wind speeds in the Venusian upper atmosphere, as in the Earth's, are of order 100 m/s (Murray *et al.* 1974). The rotation rates of the Sun and Jupiter show curious variations with latitude: the atmosphere of Jupiter may be super-rotating if the great red spot does really mark a fixed point on the surface, as suggested by Hide (1961); for the Sun super-rotation is a barren idea, because there is no solid surface for reference.

## 7. The sphere of Earth

On Earth there is a solid surface for reference, full of fascinating features visible from orbit (Bodechtel & Gierloff-Emden 1974), but sometimes half hidden by beautiful cloud forms (see frontispiece). You notice how the climate controls the past evolution and present activities of the creatures of Earth (Lamb 1972), but you realize that something so grand and complex as the weather demands more than summary treatment. So you are content to slip silently through the lower regions of the atmosphere as you zoom in to touch down on the 'sure and firm-set Earth', where you may again admire those clouds, more graceful than ever when seen from the ground (Scorer 1972).

But now you fix your gaze on the surface of Earth. You may wish to reflect on the beauty and delight of the landscape, the plants and the creatures, which natural forces and evolution have moulded into a web of interconnected life and death, a source of endless fascination to all friends of the Earth: 'the poetry of Earth is never dead' as Keats (1817) put it. You may wish to savour the almost unbearable beauty of a bright May morning in the country, or to trace how human beings have evolved in the wild, in the depths of nature, so that some never feel truly free within four walls: 'the country habit has me by the heart' (Sackville-West 1926). These wishes would take you into the realms of biology, ecology and poetry.

Instead I shall have to ask you to return to physical science, and to cast yourself back a million years to relive the fascinating saga of the gradual discovery of the Earth's shape. Imagine yourself as a primitive human being, perhaps hunting or food gathering in the wide open spaces of East Africa, with the world and the sky spread out before you. Would you have been clever enough to guess that the world was spherical? Probably not.

You might see that the Sun and Moon appear circular and decide they were disk-shaped; you might even guess they were spherical if you fathomed the reasons for the phases of the Moon. But you would need a strong effort of imagination to convince yourself that the Earth was also spherical, because after all it does *look* basically flat to someone on the surface, and the stone-ager could not see it from outside. If you were really clever, you might notice that the Sun, Earth and Moon were in line at a lunar eclipse, and say 'Ah, that circular shadow on the Moon is the Earth's shadow; so the Earth must be spherical'. Or you might notice that islands in large lakes, invisible from the shore, can be seen from a nearby hill, and say: 'Ah, the water surface is curved'. But most of the stone-age people no doubt just assumed the Earth was flat and went on for ever. The first step forward probably came with primitive religions: however absurd their world pictures may have been, many of them had an Earth of finite size set amongst the vastness of the universe, which was better than an indefinitely continuing Earth. One example is the Hindu concept of an Earth held up by a number of elephants, themselves standing on the back of a huge turtle. Earthquakes were felt when the elephants faltered a little under the Earth's weight, as well they might; and if the turtle staggered, the consequences would presumably be even more disastrous. Although this picture was very wrong, it was a picture of a bounded Earth, which transforms easily into a spherical one.

The idea that the Earth was a sphere probably developed in many cultures, and it certainly arose among the Greeks in the sixth century B.C., particularly among Pythagoras and his followers (Dreyer 1953*a*). Their ideas sprang from the philosophical concept of the sphere as a perfect shape, backed up by observation of the circular Sun and Moon, and the belief that heavenly bodies had perfect shapes. However shaky its basis, the idea was good; and it was taken up by the influential Greek philosophers like Plato (Bury 1966) and Aristotle (Guthrie 1960), to become part of the general occidental culture. The ancient Chinese, however, seem to have had a strong anti-Pythagorean prejudice that the Earth was square, though many of them were sceptical of this tradition (Needham 1959).

The most remarkable geodetic achievement of ancient times came in the third century B.C. when Eratosthenes of Alexandria measured the circumference of the Earth and probably (Dreyer 1953*b*) came within 1 % of the correct value, which is just over 40000 km. He noted that the Sun, when overhead at Aswan, was 7.2°, or $\frac{1}{50}$ of a circle, away from the vertical at Alexandria, which was almost due north. So the Earth's circumference was 50 times the distance from Aswan to Alexandria, and this distance he cunningly calculated by multiplying the average speed of a camel by the time it took on the journey. This operation has all the marks of genius: the correct concept, the correct observations, the economy of effort achieved by bringing in the camels instead of trying to stretch tape-measures across the desert, and the interdisciplinary approach – using biology to help physics.

The idea of a spherical Earth prevailed throughout the Middle Ages, which was just as well since so many scientific ideas at that time were completely wrong. Dante makes a spherical Earth and the spheres of heaven the very basis of his poem the *Divina Commedia*. The idea pervades Shakespeare's imagery, too, for example in *A Midsummer Night's Dream*, where he makes Puck orbit the Earth in 40 minutes, an estimate which was out by a factor of only 2.2 – not too disastrous an error for a quantity on the boundary of knowledge in geophysics.

## 8. Flattening the sphere

Shakespeare also foreshadows the next advance when he makes King Lear cry out 'Strike flat the thick rotundity o' the world' (Shakespeare 1608). That is just what was needed, a flattening of the poles. The Earth's polar diameter is about 43 km shorter than the equatorial diameter, so that the flattening is 43 km in 12700 km or about 1 part in 300. The accepted modern value is 1 part in 298.25.

The first numerical estimate of the flattening came from the fertile brain of Sir Isaac Newton. In his *Principia*, Newton (1687) imagined one tube of water, or canal as he called it, running from the North Pole to the Earth's centre, and another from the centre to a point on the equator. Since water shows no tendency to run from equator to pole or *vice versa* over the surface, the two canals must 'balance': gravity is slightly less in the equatorial canal because of the centrifugal effect of the Earth's rotation; so the equatorial canal needs to be slightly longer. Newton estimated the flattening as 1 part in 230. This value is rather too large because he did not allow for the increase in density towards the Earth's centre, as he realized. But for nearly a century it remained the best and most soundly based estimate.

In the early eighteenth century there was passionate interest in the shape of the Earth. The French led the way in measuring the flattening, using an extension of the method of Eratosthenes. If the Earth is flattened, the length of 1° of latitude will be greater at the poles than at the equator – not by much, 111.7 km against 110.6 km, but enough to be measured. This method was applied by J. D. Cassini and his son Jacques, who measured arc lengths in northern and southern France. Unfortunately they came to the conclusion that the Earth was *not* flattened, but drawn out at the poles like a lemon or an egg (Cassini 1720).



PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

These results emerged while Newton was still alive, and sparked off a fierce controversy. Was the Earth flattened or elongated at the poles? Who was right, Newton or the Cassinis? To settle the question, the French Academy of Sciences sent out two expeditions in 1736, one to Lapland led by Maupertuis, and a second to Peru, with leadership divided between La Condamine, Bouguer and Godin. Maupertuis's party suffered severe hardships, especially from mosquitoes, and Maupertuis himself was shipwrecked in the Gulf of Bothnia; but they were able to make reasonably good measurements, which showed that the length of 1° of latitude was greater in Lapland than near Paris. Voltaire duly congratulated Maupertuis on having 'flattened the poles and the Cassinis' (Todhunter 1873*a*). Unfortunately Maupertuis flattened the poles too much: his measurements were not as accurate as they might have been, and his value for the flattening, 1 part in 178, was considerably too large. The expedition to Peru suffered from divided leadership, as well as the obvious difficulties of operating among mist-swathed mountains remote from civilization: it was nearly 10 years before the principal members returned to Paris, only to quarrel violently. La Condamine complained that 'Ten years of labour in the new world were followed by as many of controversy in the old' (Todhunter 1873*b*). The measurements in Peru confirmed that the Earth was flattened at the poles, but there were several rival values for the flattening, ranging between 1 part in 179 and 1 part in 266. Newton's value of 1 part in 230 fell neatly between these extremes, as Voltaire (1738) pointed out:

> Vous avez confirmé dans ces lieux pleins d'ennui
> Ce que Newton connut sans sortir de chez lui.

These voyages through stormy seas to strange places achieved less than Newton, 'voyaging through strange seas of Thought, alone', as Wordsworth (1850) expressed it. Still it was admirable for so many eminent French scientists to face hardship in the tundra and jungle to settle so abstruse a question.

Between 1750 and 1950 the value for the flattening was steadily improved, until by 1957 the generally accepted value (Jeffreys 1952) was 1 part in 297.1. The methods used were the measurement of arc lengths, the recording of gravity in many parts of the world, the analysis of the motion of the Moon, and the precession of the Earth's axis. But in these 200 years of slow improvement no general features of the Earth's shape other than the flattening were securely established, although of course many small areas of the Earth were accurately mapped.

It is worth emphasizing at this point that the 'shape of the Earth' being discussed is the shape of the mean sea-level surface, continued under the land in a logical fashion, the surface usually called the *geoid.*

The arrival of satellites in 1957 brought two possible new methods of determining the Earth's shape: analysing the effect of the Earth's gravitational pull on satellite orbits could reveal the gravitational field; or the satellites could be treated as points in the sky for geometrical triangulation. The new picture of the general shape of the Earth came from the orbital analysis.

If the Earth were airless and spherical, a satellite orbit would remain fixed in shape and size, and with its plane in a fixed direction in space, apart from small lunisolar perturbations. Any departure of the Earth from a spherical form causes changes in the gravitational force acting on a satellite, and so its orbit departs from this simple form. By analysing the small changes in the orbit, the gravitational potential can be determined: adding the potential $\frac{1}{2}w^2r^2 \sin^2 \theta$ at co-latitude $\theta$ due to the Earth's angular velocity $w$ gives an augmented potential, one of whose equipotential surfaces is the mean sea-level surface, which can therefore be determined without any assumptions about the mass distribution in the Earth's interior (see Cook 1973).

The main effect of the Earth's flattening on a satellite orbit is to make the plane of the orbit rotate about the Earth's axis in the direction opposite to the satellite's motion, while leaving the orbit inclined at the same angle to the equator (figure 15). So the point where an eastbound satellite crosses the equator will move to the west, relative to the stars, as the days go by. The orbital plane of a close satellite rotates quite rapidly, at about 8° per day for a near-equatorial orbit, or 4° per day for an orbit inclined at 60° to the equator. You may be surprised that so slight a flattening has so great an effect. But it is reasonable that a flattening of 1 part in 300 (nearly $10^{19}$ tonnes of equatorial bulge) should deflect a satellite by up to 1° every time it covers 360°: actually the change is about $\frac{1}{2}$° per revolution for a near-equatorial orbit. Since the orbital plane rotates so rapidly, its rate of rotation can be accurately measured by allowing the rotation to build up for several months. If the plane rotates through 500° and is measured correct to 0.01°, equivalent to about 1 km in distance, the rate of rotation can be determined correct to 1 part in 50000.

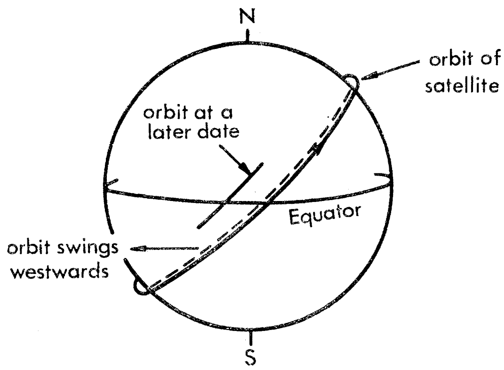



FIGURE 15. The gravitational pull of the Earth's equatorial bulge makes the orbital plane of an eastbound satellite swing westward.

This method was applied in 1958 and 1959 to *Sputnik* 2 and other satellites (Merson & King-Hele 1958; King-Hele & Merson 1959), and the accepted value for the flattening was found to be appreciably in error. The value now established is 1 part in 298.25, so that the equatorial diameter exceeds the polar diameter by 42.77 km, which is about 170 m less than was previously thought. This is not much in practical terms: it alters the length of a journey from the pole to equator and back by only 500 m. But the revision was important for both geophysics and geodesy: for geophysics because the better accuracy showed the flattening was appreciably different from the hydrostatic value appropriate for a liquid Earth, 1 part in 299.7 (Khan 1973); and for geodesy because some measurements were already accurate to 5 m, and a basic spheroid which was in error by over 100 m was unacceptable.

## 9. The pear-shaped slice through the polar ice

The improved values for the flattening obtained in 1958–9 were of course only the first step in the study of the Earth's shape by analysis of satellite orbits. The gravitational field, and hence the shape, can be regarded as the sum of an infinite series of 'zonal harmonics'. Explicitly, the longitude-averaged potential $\bar{U}$ at an exterior point distant $r$ from the Earth's centre at co-latitude $\theta$ may be written

$$\bar{U} = \frac{\mu}{r}\left\{1 - \sum_{n=2}^{\infty} J_n\left(\frac{R}{r}\right)^n P_n(\cos\theta)\right\}, \tag{8}$$

where $\mu$ is the gravitational constant for the Earth (398601 $\text{km}^3\ \text{s}^{-2}$), $R$ is the Earth's equatorial radius (6378.14 km), $P_n(\cos\theta)$ is the Legendre polynomial of degree $n$ and argument $\cos\theta$, and the $J_n$ are constant coefficients which have to be determined. The first harmonic ($n = 1$) is missing if the origin is at the Earth's centre of mass. As figure 16 shows, the second harmonic ($n = 2$) corresponds to a flattening of the sphere; the third harmonic corresponds to a triangular or pear shape; the fourth is square-shaped; the fifth has five petals; and so on. The value of $J_2$ is of order $10^{-3}$; the values of $J_3$, $J_4$, $J_5$, ... are of order $10^{-6}$.

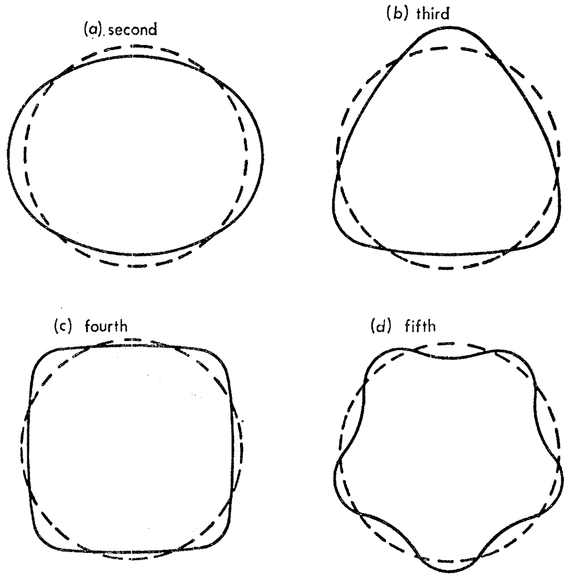



FIGURE 16. Form of the second–fifth harmonics (not to scale).

To evaluate the infinite sequence of coefficients $J_n$ is beyond human skill, and in practice the series in equation (8) has to be truncated, and the $J_n$ are evaluated only up to a chosen limit, say $n = 20$. Alternatively a selected set of coefficients may be evaluated, omitting those which seem to have little influence. This truncation is the weakest feature of the method, but it does have the self-regulating safeguard that the error is gradually reduced as time goes on and the determinations are carried to higher degree.

The even harmonics may be evaluated by analysing the rotation of the orbital plane about the Earth's axis for a number of satellites at different inclinations. If $\Omega$ is the right ascension of the ascending node (where the satellite crosses the equator going north, figure 17), the rate of rotation of the orbital plane may be written

$$\dot{\Omega} = -\left(\frac{\mu}{a^3}\right)^{\frac{1}{2}}\left(\frac{R}{p}\right)^2 \cos i\left\{\tfrac{3}{2}J_2 - \tfrac{15}{4}J_4\left(\frac{R}{p}\right)^2(1-\tfrac{7}{4}f)(1+\tfrac{3}{2}e^2) + O(J_6, J_8, \ldots) + O(J_2^2) + O(eJ_3\sin\omega, \ldots)\right\} + \text{lunisolar perturbations}, \tag{9}$$

where $a$ is the semi major axis of the orbit, $e$ is the orbital eccentricity, $i$ is the inclination, $\omega$ is the argument of perigee (see figure 17), $p = a(1-e^2)$ and $f = \sin^2 i$. Explicit forms for the $J_6$ and $J_8$ terms are given by Merson (1962) and Gooding (1966). The terms from $O(J_2^2)$ onwards in equation (9) can be pre-computed with adequate accuracy because they are small for an orbit of low eccentricity ($e < 0.1$) averaged over one or more cycles of $\omega$.

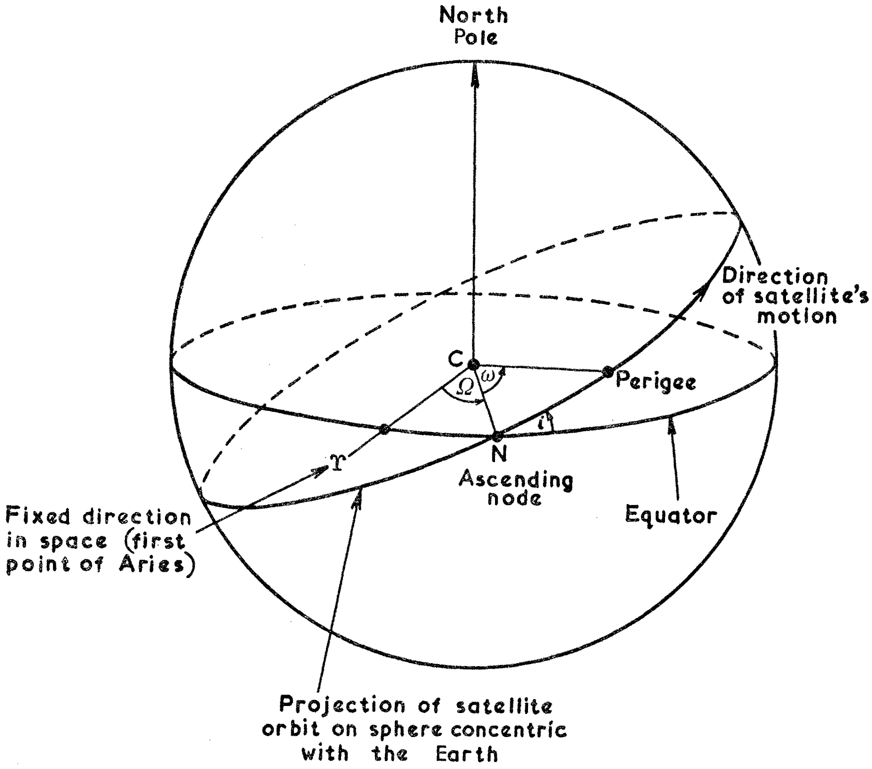



FIGURE 17. Definition of orbital elements $i$, $\Omega$, $\omega$.

For particular satellites A, B, C, . . ., say, the observed values of $\dot{\Omega}$ can be inserted to give equations of the form

$$\left.\begin{aligned} &J_2+A_4J_4+A_6J_6+\ \ldots\ = A_0 \quad \text{for satellite A,}\\ &J_2+B_4J_4+B_6J_6+\ \ldots\ = B_0 \quad \text{for satellite B,}\\ &J_2+C_4J_4+\ \ldots \quad \text{and so on.}\end{aligned}\right\} \tag{10}$$

Equations (10), in which the $A_n$, $B_n$, . . . can be regarded as constants, provide a set of simultaneous equations for the geopotential harmonic coefficients $J_2$, $J_4$, $J_6$ . . . Since the $A_n$, $B_n$, . . . depend primarily on inclination through the factor $f = \sin^2 i$, the orbits must be well spaced in inclination to avoid ill-conditioned equations. The even zonal harmonics also control the rate of rotation of the major axis, $\dot{\omega}$, and observational values of $\dot{\omega}$ can be used to obtain sets of equations similar to (10), with different numerical coefficients. However, $\dot{\Omega}$ can often be determined more accurately than $\dot{\omega}$. Thus the even harmonics $J_2$, $J_4$, $J_6$ . . . are determined from $\dot{\Omega}$ and, with lesser weight, $\dot{\omega}$.

The odd zonal harmonics $J_3$, $J_5$, $J_7$ . . ., which are antisymmetrical about the equator, have the effect of altering the distance of the perigee point from the Earth's centre. Because of the change in $\omega$, perigee swings from northern to southern hemisphere and back every few months.

For a satellite at 45° inclination, figure 18, the perigee is about 10 km further from the Earth's centre when at its southernmost limit $P_1$ than when at its northernmost, at $P_2$. This effect was first detected in 1958 with the *Vanguard* 1 satellite, and O'Keefe, Eckels & Squires (1959) deduced that the Earth had a slight tendency, then estimated as about 30 m, towards a pear shape with the stem at the north.

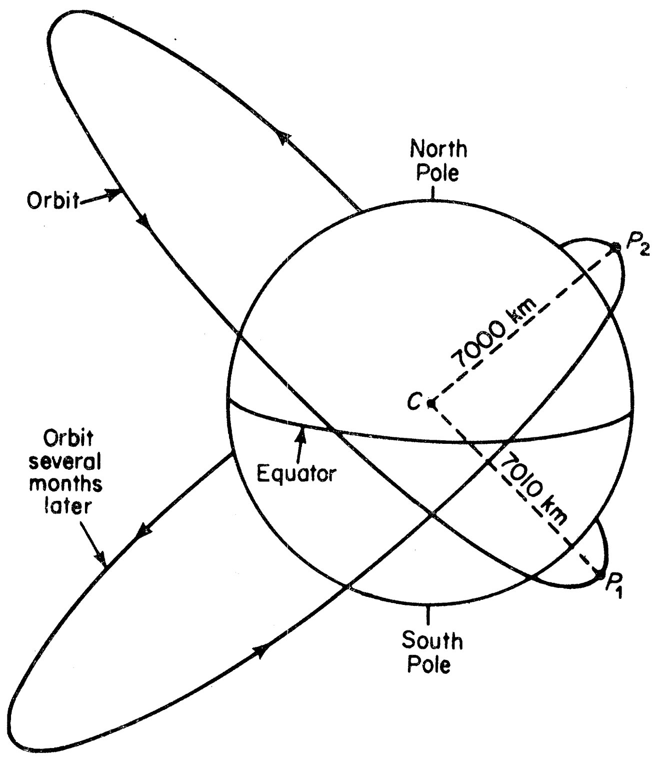



FIGURE 18. The 10 km variation in perigee distance for an orbit at 45° inclination.

The variation of perigee distance $q = a(1-e)$ due to the odd zonal harmonics may be written

$$\frac{q-q_0}{R} = \frac{\sin i}{2J_2}\left\{J_3 - \frac{2}{4-5f}\sum_{n=2}^{\infty} F_{2n+1}\, J_{2n+1}\right\}\sin\omega$$
$$+ O\{(J_2,\ J_4/J_2,\ \ldots)\ e\sin^2\omega\} + \text{lunisolar and air-drag effects,} \qquad (11)$$

where the $F_{2n+1}$ are known functions of $i$ and (to a lesser extent) $a$ and $e$. If air drag is small and $0.01 < e < 0.3$, the lunisolar, $eJ_2$, $eJ_4/J_2$ and drag terms may be calculated with adequate accuracy, leaving the observed amplitude of the oscillation in $q$, usually of order 10 km, to be equated with the factor multiplying $\sin\omega$. Thus, for each orbit analysed, a linear equation between $J_3$, $J_5$, $J_7$ . . . similar to (10) may be found, and solved for the coefficients if the orbits are at different inclinations. Other orbital elements, $i$, $\Omega$ and $\omega$, undergo similar sinusoidal oscillations, and yield equations similar in form though usually less accurate.

Sets of even and odd zonal harmonic cofficients found recently by these methods are given in table 1, overleaf.

The profile of the pear-shaped geoid given by these values is shown as a full line in figure 19 relative to a spheroid of flattening 1 part in 298.25 (broken line). The vertical scale is greatly

TABLE 1. VALUES OF ZONAL HARMONIC COEFFICIENTS

($J_2$, $J_4$ . . . from Wagner (1973); $J_3$, $J_5$ . . . from King-Hele & Cook (1974).)

| | | | |
|---|---|---|---|
| $10^9\ J_2$ | $1082635 \pm 11$ | $10^9\ J_3$ | $-2531 \pm 7$ |
| $J_4$ | $-1600 \pm 12$ | $J_5$ | $-246 \pm 9$ |
| $J_6$ | $530 \pm 26$ | $J_7$ | $-326 \pm 11$ |
| $J_8$ | $-200 \pm 29$ | $J_9$ | $-94 \pm 12$ |
| $J_{10}$ | $-224 \pm 45$ | $J_{11}$ | $159 \pm 16$ |
| $J_{12}$ | $-208 \pm 17$ | $J_{13}$ | $-131 \pm 22$ |
| $J_{14}$ | $166 \pm 25$ | $J_{15}$ | $-26 \pm 24$ |
| $J_{16}$ | $3 \pm 55$ | $J_{17}$ | $-258 \pm 19$ |
| $J_{18}$ | $-86 \pm 56$ | — | |
| $J_{20}$ | $-85 \pm 61$ | — | |

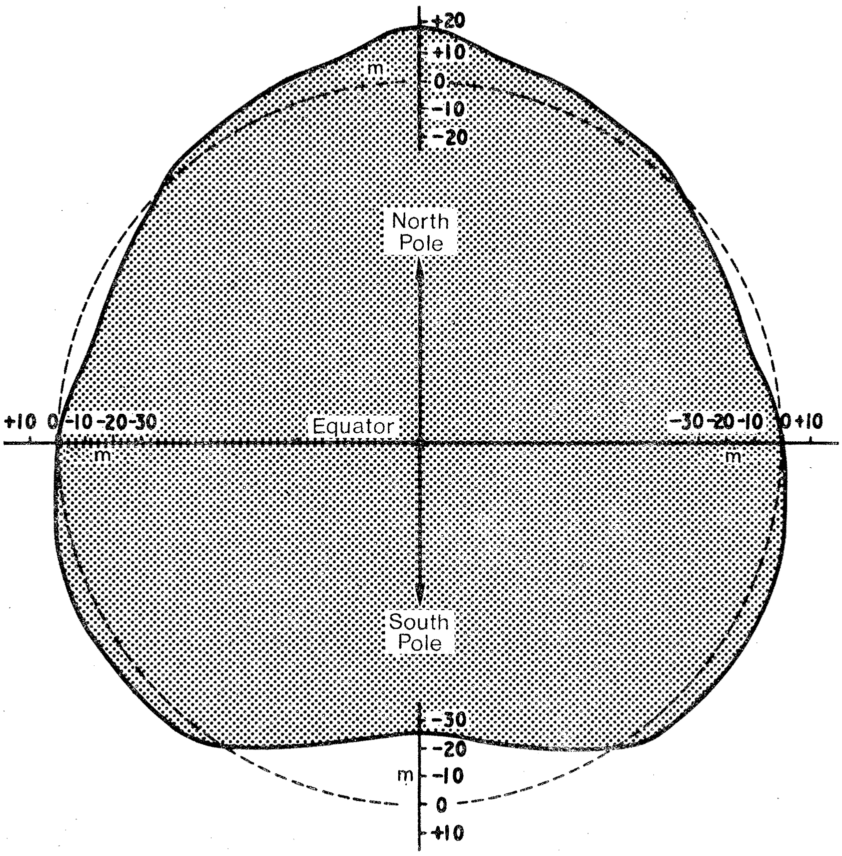



FIGURE 19. Height of the mean meridional geoid section (solid line) relative to a spheroid of flattening 1/298.25 (broken line). (With harmonics of table 1.)

exaggerated and the surface is not really concave at the South Pole. This profile, which is of course an average over all longitudes, rather than a specific physical slice, should be accurate to about 1 m. Figure 19 shows that the North Pole is 19 m above the symmetrical figure and the South Pole is 26 m below. So if you dug a hole through the ice at the North Pole and fell into the sea, you would be 45 m further from the equator than an equally stupid explorer who dug down to sea level at the South Pole.

The pear-shaped Earth came as a surprise to twentieth-century geodesists. But it was foreseen nearly 500 years ago by a man who had another curious but correct idea, that there might be some land to the west of the Atlantic Ocean – Christopher Columbus. He believed that the

Earth 'was not round in the way that is usually written, but that it is the shape of a pear that is very round, except in the place where the stem is, which is higher' (Vignaud 1911). Of course Columbus did not have a clear mental picture of the third harmonic. He imagined the Earth as basically a sphere with a pear-like stem protruding 'como una teta de muger', so he thought. Still, his anticipation of modern ideas and wording is astonishing, at a time when the discovery of the Earth's flattening was nearly 200 years ahead.

As you return towards the present, you may notice another pear-fancier, Sir Richard Phillips, the man who specified the solar atmosphere. According to George Borrow (1851), Phillips thought 'that the world is shaped like a pear and not like an apple, as the fools of Oxford say'.

## 10. 'Like a potato round the equator'

The pear-shaped polar slice is an average over all longitudes; but the variations with longitude, typified by the shape of the equator, also need to be measured. To do so, orbital analysis needs reinforcement by the use of satellites as points in the sky for triangulation. In principle, this method is simple: simultaneous camera observations of a satellite are made by, say, five observing stations; the process is repeated for satellites in other positions on other days. Each set of data yields equations between the station coordinates and the (unknown) satellite position coordinates. The latter are eliminated, leaving equations that can be solved to find the station coordinates. In practice, simultaneous observation is difficult and the geometry of the encounters is often unsatisfactory, so that the intrinsic accuracy of the observations, about 5 m, is not preserved in the solutions. Also the end-result is a number of station positions rather than a complete geoid. Such purely geometrical networks have been successfully established (Schmid 1973), but the determination of complete geoid maps has relied on combining 'broken arcs' on Earth with analysis of small orbital perturbations, which, fortunately for the analysts, mar the perfection envisaged by Browning (1864):

On the Earth the broken arcs; in the heaven a perfect round.

The procedure for a complete geoid solution is to treat the coefficients of the geopotential and the coordinates of the observing stations as parameters to be determined, and to use the observations to set up a large number of 'equations of condition' connecting the unknowns. These equations are then solved by the least-squares method. In recent solutions there have been up to 400000 equations of condition, and up to 400 unknowns.

When variations with longitude are to be studied, the geopotential $U$ is expressed as the sum of a double infinite series of 'tesseral harmonics',† which specify variations with both latitude and longitude. If $\lambda$ denotes longitude (positive to the east), the geopotential $U$ may be written

$$U = \bar{U} + \frac{\mu}{r} \sum_{l=2}^{\infty} \sum_{m=1}^{l} \left(\frac{R}{r}\right)^l P_l^m(\cos\theta)\,\{\bar{C}_{lm}\cos m\lambda + \bar{S}_{lm}\sin m\lambda\}\,N_{lm}, \tag{12}$$

where $\bar{U}$ is the longitude-averaged potential of equation (8), $P_l^m(\cos\theta)$ is the associated Legendre function of order $m$ and degree $l$, the $\bar{C}_{lm}$ and $\bar{S}_{lm}$ are constants to be determined, and $N_{lm} = \{2(l-m)!\,(2l+1)/(l+m)!\}^{\frac{1}{2}}$ is a normalizing factor. In equation (12) the suffix $l$ may be regarded as specifying *l*atitudinal variations, the suffix $m$ *m*eridional variations. For example, if all the harmonics were zero except those of fifteenth order ($m = 15$; $l = 15, 16, \ldots$) a cut

† So called in analogy with the tesserae in a Roman mosaic pavement or in the carapace of an armadillo.

along the equator (or any other latitude) would reveal a 15-petalled shape with maxima at 24° intervals in longitude.

There are other possible representations of the geopotential, e.g. as buried masses (Weightman 1967; Balmino 1973), or as a surface layer (Koch 1972); but the harmonic representation, (12), is most convenient for studies with satellites (Kaula 1969).

The variations of gravity with longitude usually produce only very small orbital perturbations, because the satellite samples all longitudes impartially and the perturbation effects tend to cancel out. Still, the amplitudes of some of the perturbations can amount to as much as 500 m along-track, with periods near 12 h or 24 h, and so are measurable with camera observations accurate to 2 seconds of arc, equivalent to 10 m at a distance of 1000 km. Alternatively, simultaneous observations by cameras (or lasers) can be used in a purely geometric way to define the $(x, y, z)$ coordinates of the observing stations relative to the Earth's centre. In practice all the data, including the Earthbound gravity measurements, are combined into one huge set of equations from which the best fit is obtained by the method of least-squares. The essential ingredients are:

(*a*) the camera (or laser) observations, with error estimates;

(*b*) the station coordinates $(x, y, z)$, regarded as unknowns, or constrained to a specified height above the geoid, with an error estimate;

(*c*) measurements of surface gravity round the world, with error estimates;

(*d*) a gravitational field model with zonal harmonics and a full set of tesseral harmonics up to perhaps degree and order 15, i.e. over 200 harmonics with coefficients to be evaluated;

(*e*) an orbital model incorporating all significant perturbations due to this set of harmonics;

(*f*) the simple equation linking the gravitational field with the sea-level (geoid) shape.

This leaves the details veiled in obscurity, but it is clear how the items are interlinked: the camera observations are made from a point at a known height above the geoid given by the harmonics, which must fit the gravity measurements, to a point on the orbit consistent with the same harmonics; the model is made as self-consistent as possible by minimizing the sums of squares of the (weighted) residual errors.

The first results using these methods were obtained from observations with the Baker–Nunn cameras of the Smithsonian Astrophysical Observatory by Izsak (1961) and Kaula (1963). Since then, observations of improving accuracy have been steadily building up. Camera observations have continued in many parts of the world, including those from the highly accurate Hewitt cameras at Malvern and Edinburgh in the U.K. Radio Doppler measurements of transmissions from satellites have provided a largely independent body of data, and gravity survey measurements have been most valuable in providing finer detail than can be seen with satellite data alone. In the last few years the camera observations have been partly supplanted by laser tracking of satellites carrying corner reflectors. The laser measurements are purely of the range to the satellite, and their accuracy has improved from about 1 m in 1970 to about 30 cm in 1974, with the potential for further improvement.

An outstanding achievement in defining the geoid was the Smithsonian Standard Earth II (Gaposchkin & Lambeck 1971), first published in 1970, shown in figure 20. The Smithsonian Standard Earth II utilized about 100000 photographic observations, from Baker–Nunn and other cameras, and about 3000 laser observations. More than 200000 equations were solved for about 400 unknowns, including the station coordinates. Figure 20 gives the contours of the geoid relative to the best-fitting spheroid, which has a flattening of 1 part in 298.25. You can see there is a depression south of India 113 m deep, and a hump 81 m high near New Guinea. This means that if you went for a swim along the equator from south of India to north of New Guinea you would increase your distance from the Earth's centre by about 190 m, although of course you would never go 'uphill'. The other main humps are about 60 m high, centred near Britain

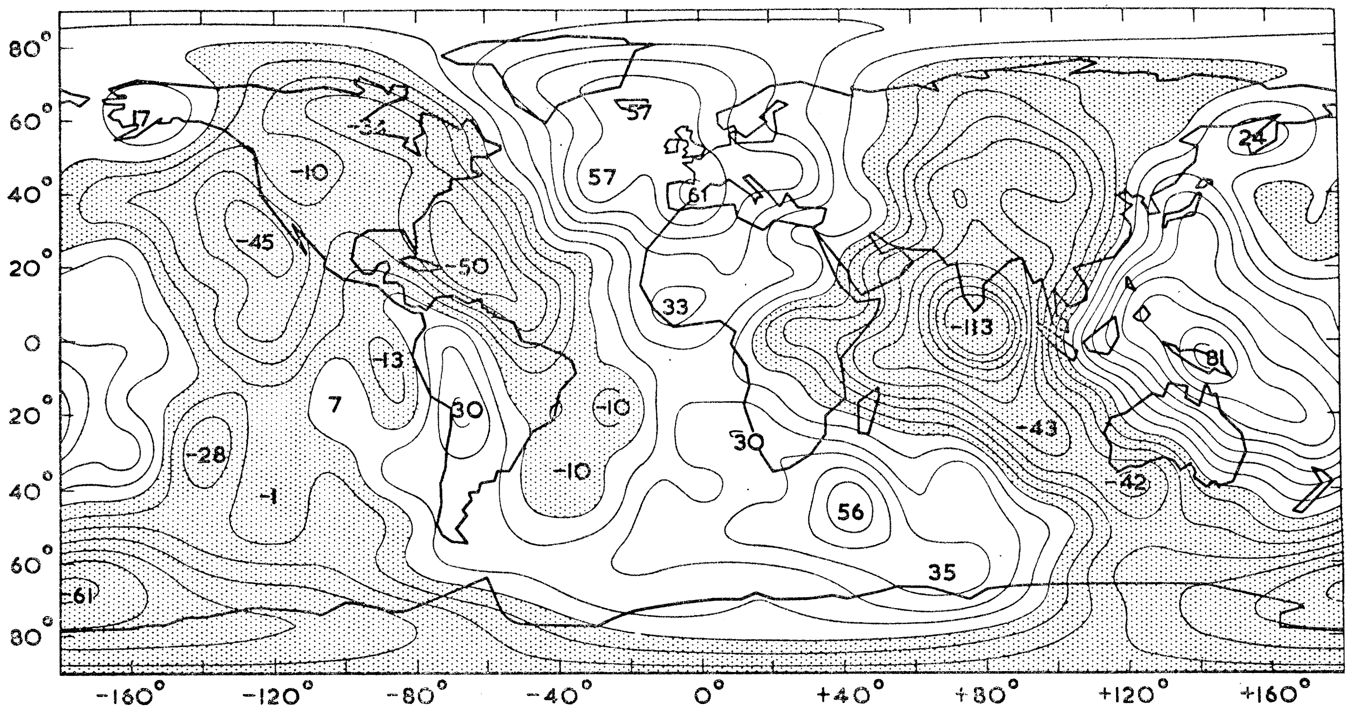



FIGURE 20. Geoid of Smithsonian Standard Earth II. Contours at 10 m intervals, relative to a spheroid of flattening 1/298.25.

and south of Madagascar. The other major depressions are south-east of New Zealand, and off the coasts of California and Florida, all approximately 50 m deep. In figure 20 the low areas are shaded dark, and if you look carefully at these areas you can see how the western hemisphere has been taken over by a goat which is deep in discussion with a man from the East, a high-brow whose cranium dominates Asia. This is not a joke, but a good way of remembering the shape of the Earth; and it is worth remembering because it is now quite well established, and in some ways is just as important as the outlines of the continents.

The slice through the equator promised in the title of this section is shown in figure 21 as an unbroken line, referred to a conventional circular equator, shown as a broken line. The vertical scale is greatly exaggerated for clarity. The depression in the Indian Ocean and the hump near New Guinea show up as the major features, and bring out more clearly the illusory uphill struggle of a swimmer or boat going from south of India to north of New Guinea. The general shape is perhaps rather like a potato, which is convenient for those with poor memories, because it nearly rhymes with equator.

Recent geoid maps derived using more accurate observations made after 1970 include the Smithsonian Standard Earth III (Gaposchkin, Williamson, Kozai & Mendes 1973), the Goddard Earth Model 4 (Lerch *et al.* 1972) and the U.S. Defense Department's World Geodetic System 1972 (Seppelin 1974). Of these the S.S.E. III and G.E.M. 4 rely largely on camera and

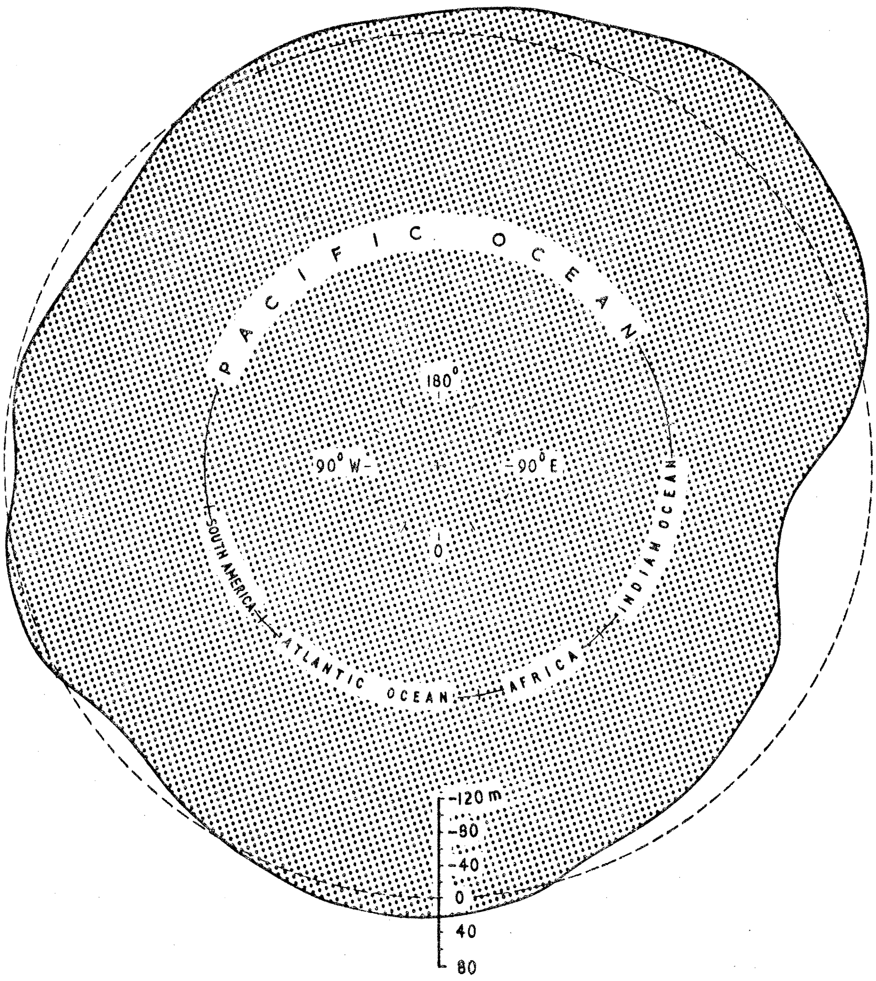



FIGURE 21. Equatorial section of geoid (solid line) relative to circular reference equator (broken line). Smithsonian Standard Earth II.

laser observations, backed up by gravimetry, while the W.G.S. 72 makes greater use of Doppler data. The most recent and detailed geoid map available is shown in figure 22: this is the G.E.M. 6, derived at the Goddard Space Flight Center (Richardson & Lerch 1974), combined with 1° × 1° gravity data. The methods used are similar to those for the previous map, but there are more observations, about 400000, with more numerous and more accurate laser observations; and the detail has been filled in with the gravity survey measurements. You can see that the main features are much the same, although the heights change slightly. For example, the depression south of India is now 112 m deep instead of 113 m, and the hump near New Guinea is 78 m high instead of 81 m. The high area near Britain has now shifted to south of Iceland, along the mid-Atlantic ridge, and the height has increased from 61 m to 68 m. But there is much more detail, and it is fascinating to see how some of the geographical features are beginning to show up clearly. There is complex structure around the Himalayas for example, and the Puerto Rico trench shows up as a minimum. This map should be accurate to 2 or 3 m in most parts of the world, and an accuracy of 1 m is confidently expected in future geoid maps, after the weaving of a web of distance measurements from sky-pointing lasers sited world-wide:

> Then weave the web of the mystic measure,
> From the depths of the sky and the ends of the Earth,

as Shelley (1820*a*) was not far wrong in saying.

## 11. The virtues of resonance

The comprehensive geoid maps are a fine achievement of measurement and mathematics. But the evaluation of some 400 coefficients from 400000 equations is a mammoth computing task, and, however great the number-crunching power may be, the inversion of immense matrices inevitably involves some nasty correlations between individual terms, which damages the accuracy of the results. What is badly needed is a method for determining accurately the coefficients of one particular order, so as to strengthen the floppy matrix.

Perhaps the most promising method for evaluating coefficients of a particular order is to take advantage of satellite orbits that exhibit resonance, and particularly fifteenth-order resonance. This is an elaborate way of saying that the track of the satellite over the Earth repeats after 15 revolutions: while the satellite goes round once – in one orbital period – the Earth spins through 24° relative to the orbital plane. So the Earth spins through 360° after 15 revolutions, and the track repeats itself. The harmonics of order 15 in the gravitational field can be regarded as having 'humps' every 24° in longitude, so their effect on a resonant orbit will be the same on each revolution, and their influence will build up day after day, while the effects of other harmonics tend to cancel out. The values of fifteenth-order harmonics can be found by measuring the magnitude of the effects they cause, as first shown by Gooding (1971). Resonance of thirteenth or fourteenth order could also be used, but fifteenth-order resonance proves most useful, because a fifteenth-order resonant satellite has an orbital period near 95 min, which corresponds to a satellite height of about 500 km, where air drag is quite appreciable: consequently several satellites each year experience fifteenth-order resonance as they decay slowly in near-circular orbits under the influence of drag.

The inclination $i$ of the orbit to the equator is the orbital parameter which most clearly registers the effects of fifteenth-order resonance. The theory of such resonances has been developed by Allan (1967, 1973), and the rate of change of $i$ in a near-circular orbit due to the effect of fifteenth-order resonant terms may be written

$$\frac{\mathrm{d}i}{\mathrm{d}t} = nG(R/a)^{15}\Big\{\bar{C}_{15}\sin\Phi - \bar{S}_{15}\cos\Phi + \text{terms in } (\bar{C}_{30}, \bar{S}_{30}) \begin{matrix}\cos\\ \sin\end{matrix}\, 2\Phi;\ (\bar{C}_{45}, \bar{S}_{45}) \begin{matrix}\cos\\ \sin\end{matrix}\, 3\Phi;\ \text{etc.} + \text{terms of order } 10e\ (\bar{C}_{l,15}, \bar{S}_{l,15}) \begin{matrix}\cos\\ \sin\end{matrix}\, (\Phi \pm \omega),\ \text{etc.}\Big\} \tag{13}$$

In equation (13), $n$ is the mean motion ($=\dot{M}$), and

$$G = 0.5877(15-\cos i)\,(1+\cos i)\,\sin^{13} i. \tag{14}$$

The resonance angle $\Phi$ is given by

$$\Phi = \omega + M + 15\,(\Omega - \nu), \tag{15}$$





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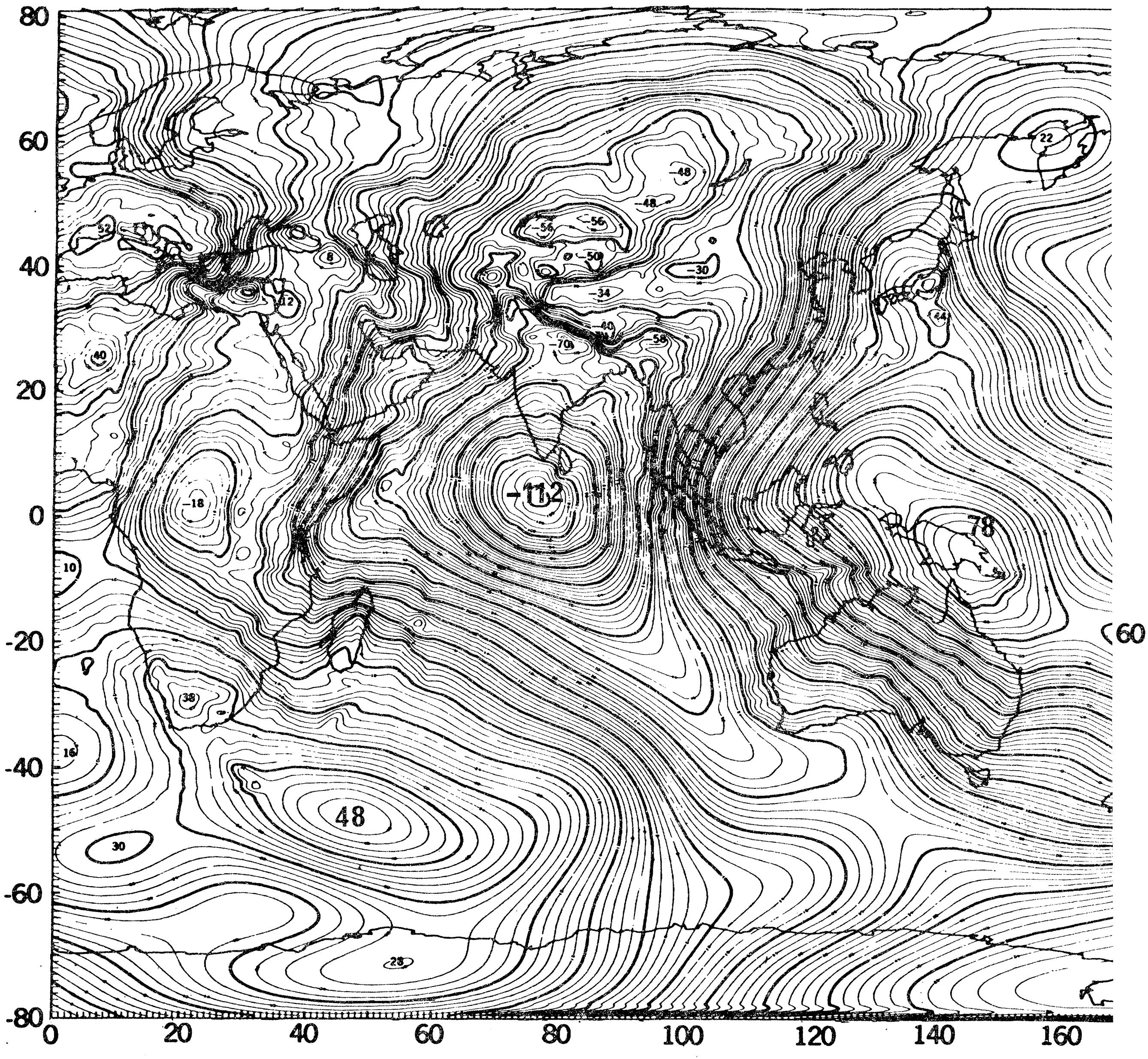


FIGURE 22. Geoid map combining G.E.M. 6 Earth model and 1° × 1°



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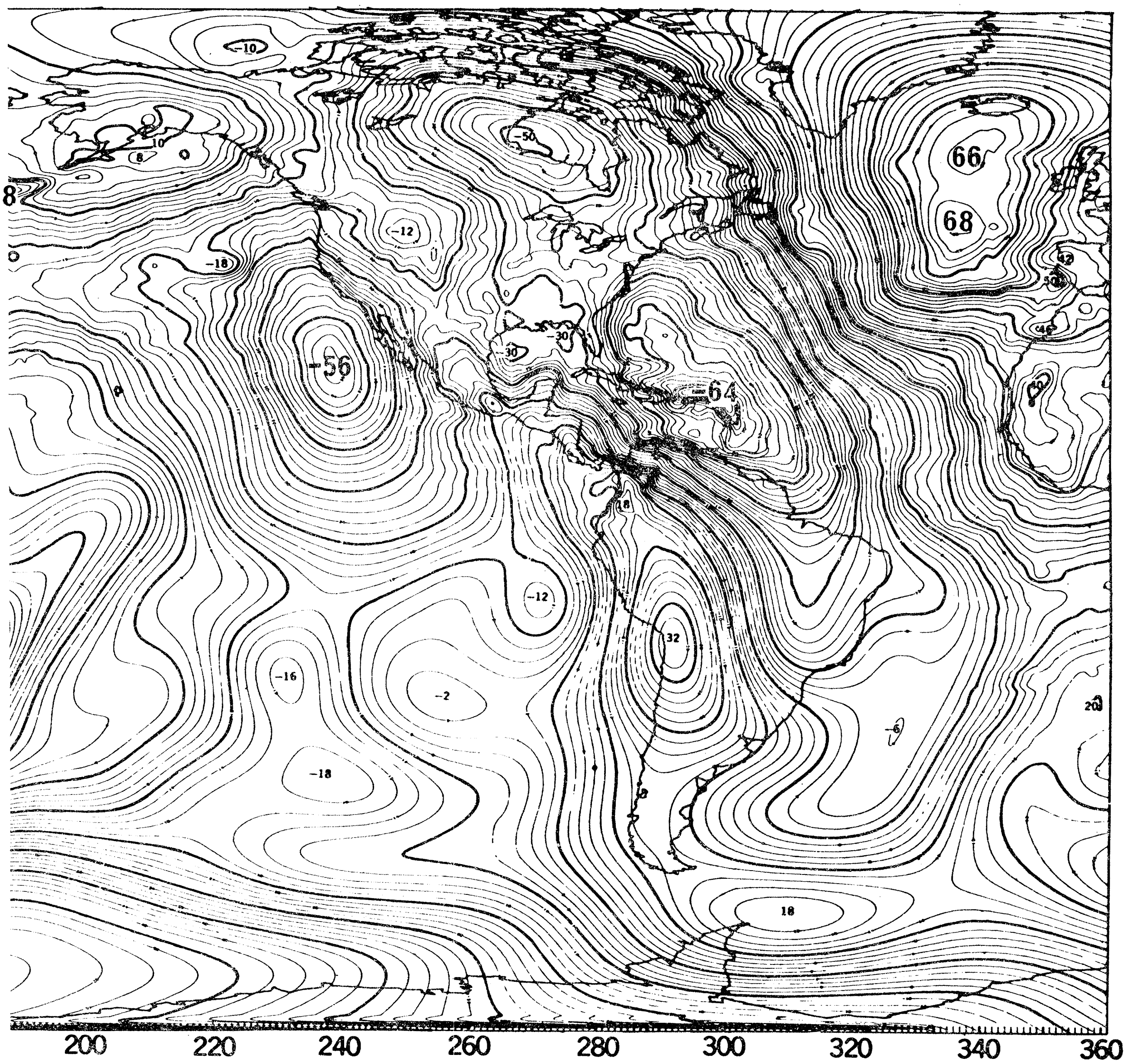


Data. Relative to a spheroid of flattening 1/298.255. Heights in metres.



PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

where $M$ is the mean anomaly and $\nu$ the sidereal angle: exact fifteenth-order resonance occurs when $\dot{\Phi} = 0$. The quantities $\bar{C}_{15}$, $\bar{S}_{15}$, $\bar{C}_{30}$ . . . are constants to be determined (if they are significant) and theory shows that they may be expressed as linear functions of the individual coefficients $\bar{C}_{l,15}$, $\bar{S}_{l,15}$, $\bar{C}_{l,30}$, . . . . For example,

$$\bar{C}_{15} = \bar{C}_{15,15} + Q_{17}\bar{C}_{17,15} + Q_{19}\bar{C}_{19,15} + Q_{21}\bar{C}_{21,15} + \ldots, \tag{16}$$

where the $Q_l$ are coefficients dependent on the orbital inclination, specified by Allan (1973).

In fitting the theoretical equation (13) to the observed variation of $i$, the best results are likely to be obtained if the simplest model can be used, by omitting the terms in $e$, of which only the first four are specified in equation (13): there are various other terms in $\cos(k\Phi \pm q\omega)$ or $\sin(k\Phi \pm q\omega)$ that may be significant, where $k$ and $q$ are integers $\geqslant 1$. Since the $e$ terms have a factor of order 10, orbits with $e$ less than about 0.003 are likely to give the best results. The expected magnitude of a typical $\bar{C}_{lm}$ coefficient is $10^{-5}/l^2$ (Kaula 1966), so the 30th and 45th order terms are likely to be considerably smaller than the fifteenth-order terms, but may still sometimes have a significant effect.

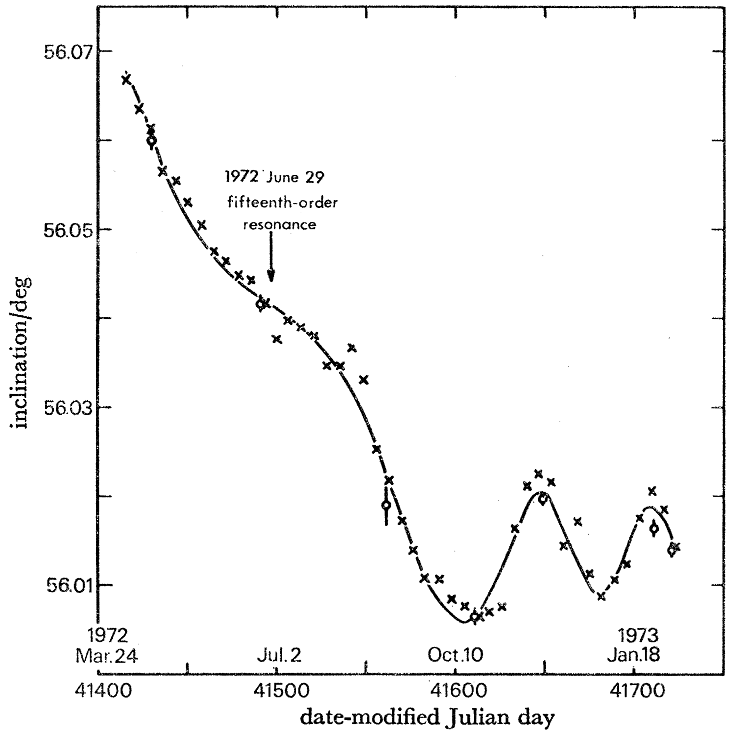


FIGURE 23. *Cosmos* 72, 1965–53B: values of inclination at resonance, with fitted theoretical curve. ×, U.S. Navy values, assumed s.d. 0.003°; ɸ, values from R.A.E. orbits, with s.d.; ——, fitted theoretical curve.

Figure 23 shows the result of fitting the variation in inclination for *Cosmos* 72, a Russian satellite launched in 1965, which passed through fifteenth-order resonance in 1972. The orbit was nearly circular ($e = 0.003$), and the large decrease in inclination, nearly 0.07°, is fitted well by the theoretical curve, giving

$$10^9\bar{C}_{15} = -236 \pm 4; \quad 10^9\bar{S}_{15} = -109 \pm 9. \tag{17}$$

The inclusion of $\bar{C}_{30}$ and $\bar{S}_{30}$ did not improve the fit, and did not significantly alter the values of $\bar{C}_{15}$ or $\bar{S}_{15}$ (King-Hele 1974*b*).

A value of $\bar{C}_{15}$ from one satellite gives one equation of the form (16) linking the individual coefficients of order 15 and degree 15, 17, 19, 21, .... Values of $\bar{C}_{15}$ and $\bar{S}_{15}$ from satellites at a variety of inclinations therefore give a set of simultaneous equations, which can be solved for the individual coefficients of order 15 and degree 15, 17, 19, 21, .... From an analysis of 11 resonant orbits at inclinations between 30° and 90°, King-Hele, Walker & Gooding (1974*a*) obtained the values given in the first three columns of table 2 for harmonics of order 15 and odd degree up to 31.

Table 2. Values of fifteenth-order harmonic coefficients from analysis of resonant orbits

(Values from King-Hele, Walker & Gooding 1974*a*, *b*);

| $l$ | $10^9\bar{C}_{l,15}$ | $10^9\bar{S}_{l,15}$ | $l$ | $10^9\bar{C}_{l,15}$ | $10^9\bar{S}_{l,15}$ |
|---|---|---|---|---|---|
| 15 | $-21.5 \pm 0.9$ | $-8.4 \pm 0.9$ | 16 | $-13.7 \pm 1.3$ | $-18.5 \pm 2.7$ |
| 17 | $4.4 \pm 1.6$ | $9.0 \pm 1.5$ | 18 | $-42.3 \pm 1.8$ | $-34.7 \pm 3.4$ |
| 19 | $-15.6 \pm 2.6$ | $-14.1 \pm 2.7$ | 20 | $10.5 \pm 3.1$ | $29.8 \pm 5.2$ |
| 21 | $10.4 \pm 3.0$ | $7.3 \pm 3.5$ | 22 | $-8.6 \pm 3.8$ | $-20.2 \pm 7.4$ |
| 23 | $22.5 \pm 2.8$ | $1.2 \pm 4.4$ | | | |
| 25 | $-0.9 \pm 4.7$ | $-3.8 \pm 5.3$ | | | |
| 27 | $-11.2 \pm 3.3$ | $9.1 \pm 3.2$ | | | |
| 29 | $-20.5 \pm 5.4$ | $-1.2 \pm 6.1$ | | | |
| 31 | $17.7 \pm 6.6$ | $-1.0 \pm 7.1$ | | | |

Just as the variations in inclination at resonance give the fifteenth-order harmonics of odd degree, so the variations in eccentricity give those of even degree ($l$ = 16, 18, 20 ...). The main terms in the variation of $de/dt$ given by theory are of the form $\cos(\Phi \pm \omega)$ or $\sin(\Phi \pm \omega)$, but numerous smaller terms of the form $\cos(k\Phi \pm q\omega)$ or $\sin(k\Phi \pm q\omega)$ may sometimes prove significant and have to be tested in the fitting. Also the large variation in $e$ due to odd zonal harmonics has to be removed. Because of these complications the evaluation of the even-degree fifteenth-order harmonics is less advanced than for those of odd degree, as table 2 shows.

Although laser observations accurate to about 30 cm are now required if improvements are to be made in the global geoid models, these analyses of resonance, which give some individual coefficients much more accurately than the global models, do not require observations of such high precision. The values of table 2 were obtained by analysing orbits determined from photographic, radar and visual observations. The photographic observations are from the most accurate satellite cameras in the world, the Hewitt cameras at Malvern and Edinburgh, which are usually accurate to about 2″ (seconds of arc), or 10 m. But the great majority of the observations used are accurate to no better than 100″: about half are US Navy observations and the other half are quite primitive-looking, being made by skilled volunteer visual observers with stopwatch and binoculars working from their back gardens (King-Hele 1966).

Having seen the shape of the Earth and savoured the complexity of its harmonic make-up, you will probably disagree with Shelley, who called it a 'sphere of divinest shapes and harmonies'. With its bulging waistline and many blemishes, the Earth qualifies not for the line of beauty defined by Hogarth (1753), but rather for a down-to-earth Shakespearian phrase, 'changed to a worser shape thou canst not be'.

## 12. The plutonic realm

As a sceptical visitor from space, you may ask, 'why bother to measure the Earth so accurately?' The answer to this is quite easy. All down the centuries people have found the shape of the Earth a rather fascinating subject, but only as a matter 'of academic interest'. Now a great change has come: endowed with 10 cm accuracy from satellite laser tracking, geodesy is about to invade the realm of geophysics, which will soon find itself re-invigorated by an injection of exact measurement (King-Hele 1974*a*). Fact will move in and the speculators will be driven out, to exercise their undoubted talents elsewhere, probably on other planets.

In the past the accuracy of geodesy has not been significant for geophysics, because the Earth's rotational energy is so enormous, about $10^{30}$ J, that even catastrophic geophysical events have effects measured only in centimetres. For example, every major earthquake slightly redistributes the Earth's mass, and therefore alters the pole position as measured on the Earth's surface. The motion of the pole, which has been measured for many years, has an amplitude of order 10 m (Anderle 1973). But most of this is a nearly periodic variation, comprising the 14-month Chandler wobble, and other annual and semi-annual variations caused by changes in the angular momentum of the atmosphere (Melchior 1972). The features of prime geophysical interest are the minor twitches in the polar motion, amounting to less than 1 m. These probably register the net effect of tectonic movements – not only earthquakes but also possibly slow continental movements, which may proceed by jerks. Furthermore, the twitches, small as they are, may be enough to make the Earth throw its enormous weight around a little differently and this may act as the trigger for tectonic movement (Pan 1972). The Earth may be a self-exciting earthquake generator. Exact measurement of polar motion and successful analysis of the Earth's dynamics will probably be needed before tectonic movements are fully understood.

By running your time machine back merely 50 years you can see great changes in the established view of geological processes. Fifty, or even 25 years ago, the idea of continental drift was rank heresy. Alfred Wegener put forward his ideas on the mobility of the continents in 1915 (Wegener 1915, 1967), but most geologists rejected his conclusions: after all, he belonged to the wrong union, being a meteorologist and geodesist. However, a few geologists supported his ideas, including Arthur Holmes (1929), and eventually, in the course of the 1960s, heresy flipped over into orthodoxy, with the vagueness of 'continental drift' crystallizing into the relative precision of plate tectonics. The story of this 'revolution in the Earth sciences' has been nicely told by Hallam (1973). The majority of geophysicists, but not all, now see the Earth's surface as fractured into a limited number (between 10 and 20) of rigid plates, about 100 km thick, on which oceans or land (or both) ride as if over a lake of treacle. New crust is generated at ocean ridges such as the mid-Atlantic ridge, where Europe, it is thought, is being moved away from North America at a rate which over millions of years has an average of 2 cm/year. But the motion is jerky, if earth tremors are any guide, and the actual motions over 10 years should soon be directly measurable across some of the plate boundaries with the new techniques of satellite geodesy, such as laser tracking (Agreen & Smith 1973), perhaps aided by other methods such as very long baseline interferometry, satellite-to-satellite tracking and laser ranging to reflectors on the Moon.

Figure 24 shows the approximate boundaries of the main tectonic plates. Double lines indicate the boundaries where it appears that new crust is being formed, pushing the adjoining plates apart. Spiked lines denote boundaries where the main motion seems to be a swallowing-up

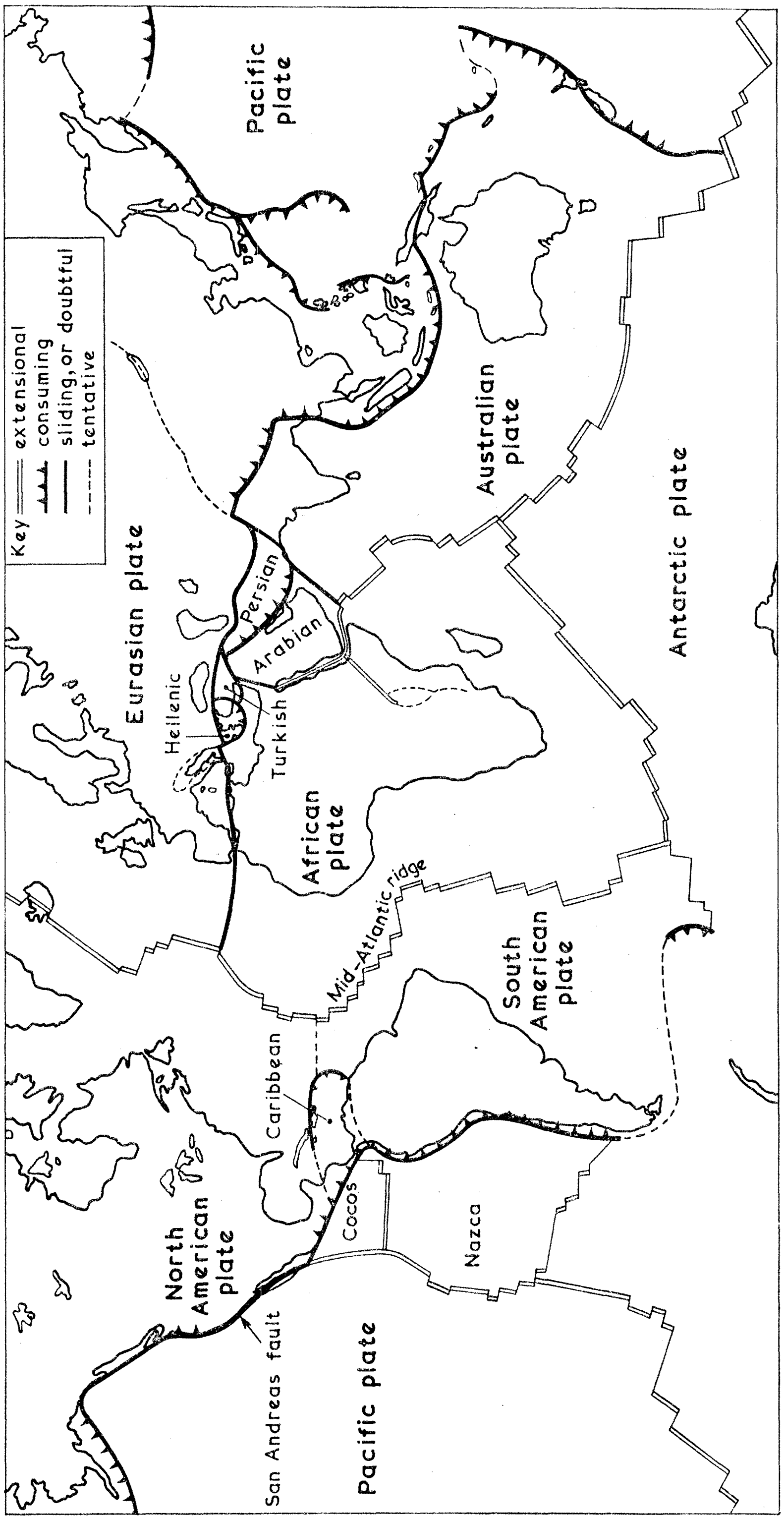


FIGURE 24. The plated Earth, showing approximate plate boundaries.

of one of the plates. There are also some boundaries where one plate slides against another, and the plain lines in figure 24 indicate either that the motion is mainly sliding or that it is unknown or complex.

The idea of rigid plates moving over a treacly plutonic base was delightfully pure and simple in the days of its youth and innocence. But, just as ordinary household plates are liable to crack and break into micro-plates when they age, so cracks are appearing in the tectonic plates now that the concepts are reaching a respectable age. Again, laser ranging to satellites from stations within one continent, particularly southern Europe, should show whether this splintering is important or not. For recent reviews of plate tectonics, see Tarling & Runcorn (1973), and Le Pichon, Francheteau & Bonnin (1973).

Measuring how fast America is separating from Europe may be more dramatic than the accurate determination of the geoid and gravitational field, but the latter may prove to be more important. The contours of the geoid, revealed in greater detail both by existing methods and by radar altimeters in satellites, will provide an illuminating guide to the geographical and geological features which they reflect. And the evaluation of high harmonics in the gravitational field will provide a crucial test of the rival theories about the structure and distribution of mass within the Earth (Allan 1972).

## 13. Envoi

Human beings depend on the Earth and Air for all their material needs. So a better understanding of Earth and Air should bring them benefit – in seeking deep new sources of the minerals they have squandered so unthinkingly; in predicting and partially controlling the weather that often dictates their activities; in predicting and perhaps preventing the earthquakes that sometimes engulf them. But if you turn back your clock you will see that these current lords of the Earth have an unfortunate habit of misusing new knowledge, gripped as they are by untamed drives appropriate to earlier phases of their evolution (King-Hele 1970).

During your brief terrestrial visit you will, like the cosmic traveller of Stapledon (1937), have taken the chance to enter into the human mind and you will perhaps have found something to admire in the fine intellectual edifice of scientific knowledge, even if you doubt whether human civilization will survive the continued exercise of the baser human passions. So perhaps you will carry with you mixed feelings of admiration and sadness as you streak away *en voie* from the Earth's pleasant pastures, soon to be seen as only a speck

Pinnacled dim in the intense inane.

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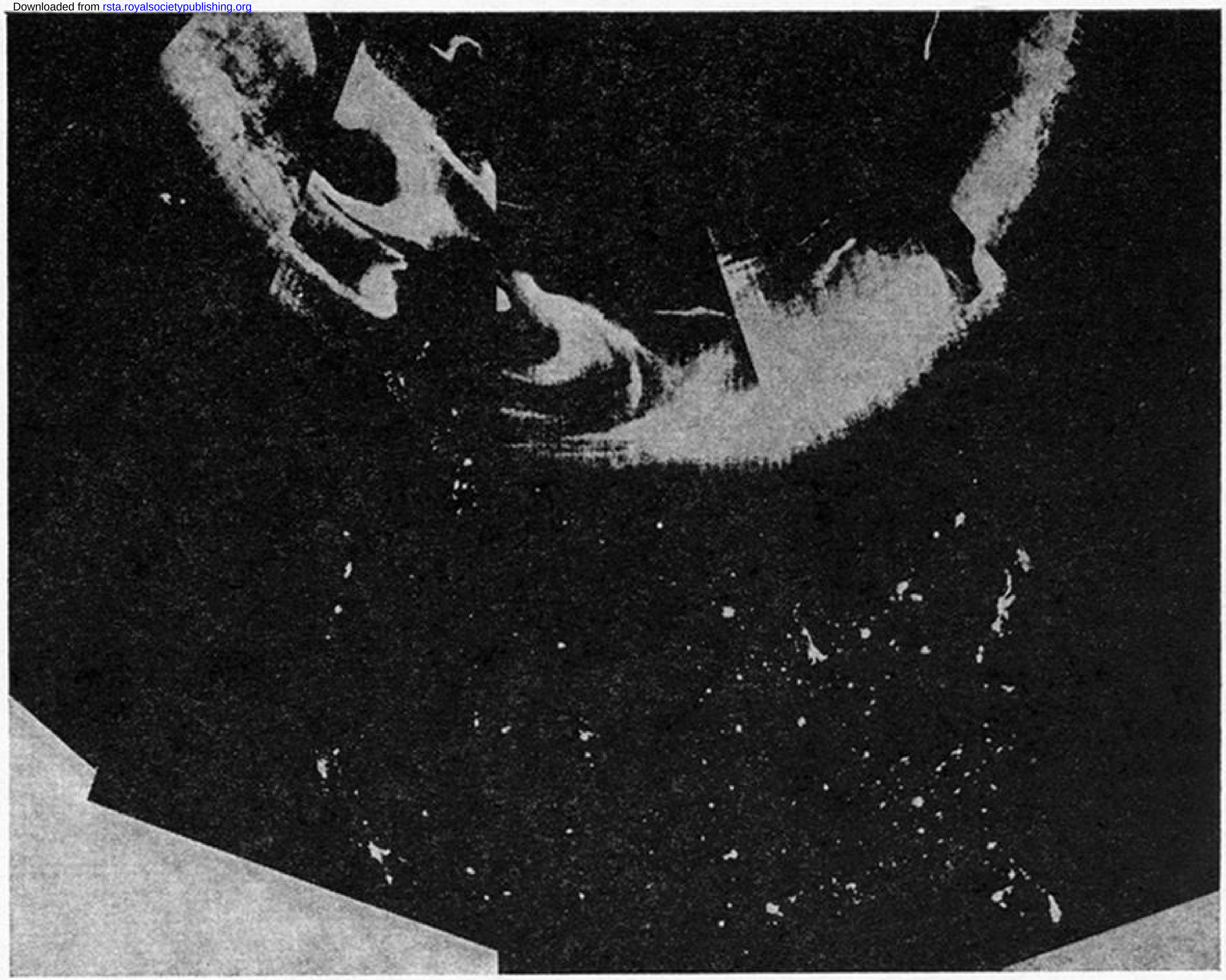
PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES





PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

A view of Earth and Air on 28 May 1974, from the NASA Synchronous Meteorological Satellite 1 at a height of 36000 km above the equator at longitude 75° W. The southern half of the U.S.A. is free of cloud, but South America is partly hidden by clouds. The west coast of Africa can be seen on the right and storms in the Atlantic at the top right.



IGURE 5. Aurora and city lights, U.S.A. and Canada, near midnight 14 February 1972. Mosaic photograph from five satellite passes: the straight lines mark the dividing lines between passes. Reproduced by permission of Dr E. H. Rogers.



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